The Dynamics of Comparative Advantage*

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Abstract

This paper characterizes the dynamic empirical properties of comparative advantage. We revisit two strong empirical regularities in international trade that have previously been studied in parts and in isolation. There is a tendency for countries to concentrate exports in a few sectors. We show that this concentration arises from a heavy-tailed distribution of industry export capabilities that is approximately log normal and whose shape is stable across 90 countries, 133 sectors, and 40 years. Likewise, there is a tendency for mean reversion in national industry productivities. We establish that mean reversion in export capability, rather than indicative of convergence in productivities or degeneracy in comparative advantage, is instead consistent with a stationary stochastic process. We develop a GMM estimator for a stochastic process that generates many commonly studied stationary distributions and show that the Ornstein-Uhlenbeck special case closely approximates the dynamics of comparative advantage. The OU process implies a log normal stationary distribution and has a discrete-time representation that can be estimated with simple linear regression.

Keywords: International trade; comparative advantage; generalized logistic diffusion; estimation of diffusion process

JEL Classification: F14, F17, C22

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1 Introduction

Comparative advantage has made a comeback in international trade. After a hiatus, during which the Ricardian model was widely taught to students but rarely applied in research, the role of comparative advantage in explaining trade flows is again at the center of inquiry. Its resurgence is due in large part to the success of the Eaton and Kortum (2002) model (EK hereafter). Chor (2010) and Costinot et al. (2012) find strong support for EK in cross-section trade data, and a rapidly growing literature uses EK as a foundation for quantitative modeling of changes in trade policy and other shocks (e.g., Costinot and Rodríguez-Clare 2014, Di Giovanni et al. 2014, Caliendo and Parro 2015).

In this paper, we characterize empirically how comparative advantage evolves over time. From the gravity model of trade, we extract a measure of country export capability, which we use to evaluate how export performance changes for 90 countries in 133 industries from 1962 to 2007. Distinct from Waugh (2010), Costinot et al. (2012), and Levchenko and Zhang (2013), we do not use industry production or price data to evaluate country export prowess.[1] Instead, we rely solely on trade data, which allows us to impose less structure on the determinants of trade and to examine both manufacturing and nonmanufacturing industries at a fine degree of disaggregation and over a long time span. These features of the analysis help us to uncover the stable distributional properties of comparative advantage, which heretofore have been unrecognized.

The gravity framework is consistent with a large class of trade models (Anderson 1979, Anderson and van Wincoop 2003, Arkolakis et al. 2012), of which EK is one example. These have in common an equilibrium relationship in which bilateral trade in a particular industry and year can be decomposed into an exporter-industry fixed effect, which measures the exporting country's *export capability* in an industry; an importer-industry fixed effect, which captures the importing country's effective demand for foreign goods in an industry; and an exporter-importer component, which accounts for bilateral trade frictions (Anderson 2011).[2] We estimate these components for each year in our data, using both OLS and methods developed by Silva and Tenreyro (2006) and Eaton et al. (2012) to correct for zero bilateral trade flows.[3] In the EK model, the exporter-industry fixed effect embodies the location parameter of a country's productivity distribution for an industry, which fixes its sectoral efficiency in producing goods. By taking the deviation of a country's log export capability from the global industry mean, we obtain a measure of a country's absolute advantage in an industry. Further normalizing absolute advantage by its country-wide mean, we remove the effects of aggregate country growth. We refer to export capability under its double normalization as a measure of comparative advantage.

[1] On the gravity model and industry productivity also see Finicelli et al. (2009, 2013), Fadinger and Fleiss (2011), and Kerr (2013).

[2] For an alternative approach to decomposing underlying sources of changes in bilateral trade, see Gaubert and Itskhoki (2015).

[3] We verify that our results are robust to replacing our gravity-based measure of export capability with Balassa's (1965) index of revealed comparative advantage. Additional work on approaches to accounting for zero bilateral trade includes Helpman et al. (2008), Fally (2012), Head and Mayer (2014).

After estimating the gravity model, our analysis proceeds in two parts. We first document two strong empirical regularities in country exporting, and then, informed by these regularities, specify a stochastic process for export advantage. Though we motivate our approach using EK, we are agnostic about the origins of country export capabilities. The Krugman (1980), Heckscher-Ohlin (Deardorff 1998), Melitz (2003), and Anderson and van Wincoop (2003) models also yield gravity specifications and give alternative interpretations of the exporter-industry fixed effects that we use in our analysis. Our aim is not to test one model against another but rather to identify the dynamic properties of absolute and comparative advantage that any theory of their determinants must explain. As we will show, these properties include a stationary distribution for comparative advantage whose shape is common across countries, industries, and time.

The first empirical regularity that we report is stable heavy tails in the distribution of country-industry exports. In a given year, the cross-industry distribution of absolute advantage for a country is approximately log normal, with ratios of the mean to the median of about 7. For the 90 countries in our data, the median share for the top good (out of 133) in a country's total exports is 23%, for the top 3 goods is 46%, and for the top 7 goods is 64%.[4] The heavy-tailedness of export advantage is both persistent and pervasive. The approximate log-normal shape applies to individual countries over time and, at a given moment in time, across countries that specialize in different types of goods.

Stability in the shape of the distribution of comparative advantage makes the second empirical regularity all the more surprising: there is continual and relatively rapid turnover in countries' top export industries. Among the goods that account for the top 5% of a country's current absolute-advantage sectors, 60% were not in the top 5% two decades earlier.[5] This churning is consistent with mean reversion in comparative advantage. In an OLS regression of the ten-year change in log export capability on its initial log value and industry-year and country-year fixed effects—a specification to which we refer compactly as a decay regression—we estimate mean reversion at the rate of about one-third per decade. Levchenko and Zhang (2013) also find evidence of mean reversion in comparative advantage, in their case for 19 aggregate manufacturing industries, which they interpret as evidence of convergence in industry productivities across countries and of the degeneracy of comparative advantage. This interpretation, however, is subject to the Quah (1993, 1996) critique of cross-country growth regressions: mean reversion in a variable is uninformative about its distributional dynamics. Depending on

[4]See Easterly and Reshef (2010) and Freund and Pierola (2013) for related findings on export concentration. Hidalgo and Hausmann (2009) and Hausmann and Hidalgo (2011) link export concentration to sparsity in the bilateral export-flow matrix. Using cross sections of Balassa comparative advantage measures for select years (1985, 1992 and 2000, or 2005) from data similar to ours but at the SITC 4-digit or HS 6-digit product levels, they document that a country's concentration in few products above a comparative-advantage threshold is positively correlated with the average "ubiquity" of the country's comparative-advantage products (where "ubiquity" is the frequency that a product exceeds the comparative-advantage threshold in any country). Our stochastic model admits such covariation in the cross section.

[5]On changes in export diversification over time see Imbs and Wacziarg (2003), Cadot et al. (2011), and Sutton and Trefler (2016).

the stochastic properties of a series, mean reversion may alternatively coexist with a cross-section distribution that is degenerate, non-stationary, or stationary. To understand distributional dynamics, one must take both the stochastic process and the cross-sectional distribution over time as the units of analysis. Our finding that the heavy tails of export advantage are stable over time suggests that, quite far from being degenerate, the distribution of comparative advantage for a country is stationary.

In the second part of our analysis, we estimate a stochastic process that can account for the combination of a stable cross-industry distribution for export advantage with churning in national industry export rankings. Our OLS decay regression provides a revealing starting point for the exercise. As a mean-reverting AR(1) specification, the decay regression is a discrete-time analogue of a continuous-time *Ornstein-Uhlenbeck* (OU) process, which is the unique Markov process that has a stationary normal distribution (Karlin and Taylor 1981). The OU process is governed by two parameters, which we recover from our OLS estimates. The *dissipation rate* regulates the rate at which absolute advantage reverts to its long-run mean and determines the shape of its stationary distribution; the *innovation intensity* scales the stochastic shocks to absolute advantage and determines how frequently industries reshuffle along the distribution. Our estimates of the dissipation rate are very similar across countries and sectors, which confirms that the heavy-tailedness of export advantage is close to universal. The innovation intensity, in contrast, is higher for developing economies and for nonmanufacturing industries, which affirms that the pace at which industry export ranks turn over is idiosyncratic to countries and sectors.

Although attractive for its simplicity, the OU is but one of many possible stochastic processes to consider. To be as expansive as possible in our characterization of export dynamics while retaining a parametric stochastic model, we next specify and estimate via GMM a *generalized logistic diffusion* (GLD) for absolute advantage. The GLD has the OU process as a limiting case, which allows us to test the linearity restrictions of the OLS decay regression and the implied assumption of log normality for export advantage. The GLD adds an additional parameter to estimate—the *decay elasticity*—which allows the speed of mean reversion to differ from above versus below the mean. Slower reversion from above the mean, for instance, would indicate that absolute advantage tends to be "sticky," eroding slowly for a country once acquired. The appeal of the GLD is its ample flexibility in describing the distribution of export advantage. The stationary distribution for the process is a generalized gamma, which unifies the extreme-value and gamma families and therefore nests many common distributions (Crooks 2010), including those used in the analysis of city size (Gabaix and Ioannides 2004, Luttmer 2007) and firm size (Sutton 1997, Gabaix 1999).[6]

Having estimated the GLD, we evaluate the fit of the model and its performance under alternative parameter restrictions. We take the GMM time-series estimates of the three global parameters—the dissipation rate, the

[6]Cabral and Mata (2003) use a similar generalized gamma to study firm-size distributions.

innovation intensity, and the decay elasticity—and predict the cross-section distribution of absolute advantage, which is not targeted in our estimation. Based on just three parameters (for all industries in all countries and in all years), the predicted values match the cross-section distributions with considerable accuracy. We also compare the observed churning of industry export ranks within countries over time with the model-predicted transition probabilities between percentiles of the cross-section distribution. The predicted transitions match observed churning, except in the very lower tail of the distribution. This exercise also allows us to compare the performance of the GLD to the OU process. While the data select the GLD over the more restrictive OU form, the two models yield nearly identical predictions for period-to-period transition probabilities between quantiles of the distribution of export advantage. This finding is of significant practical importance for it suggests that in many applications the OU process, with its linear representation of the decay regression, will adequately describe export dynamics. An OU process greatly simplifies estimating multivariate diffusions, which would encompass the intersectoral and international linkages in knowledge transmission that are at the core of recent theories of trade and growth (Eaton and Kortum 1999, Alvarez et al. 2013, Buera and Oberfield 2016, Gaubert and Itskhoki 2015).

What does it mean for comparative advantage to follow a diffusion process? In purely econometric terms, the process implies that ongoing stochastic innovations to export capability offset mean reversion and perpetually reshuffle industries along the distribution, thereby preserving the stable heavy tails in the cross section. In economic terms, it means that the dynamics of export growth are common to broad classes of industries, including manufacturing and nonmanufacturing activities that we typically think of as having distinct forms of innovation. Though countries may discover what they are good at producing in numerous ways(Hausmann and Rodrik 2003), the rise and fall of industries appear to have common patterns. In Finland, Nokia's reasearch and development in cellular technology turned the country into a powerhouse in mobile telephony in the early 2000s. For Costa Rica, it was foreign direct investment, in particular Intel's 1996 decision to build a chip factory near San Jose, that made electronics the country's largest export (Rodríguez-Clare 2001). In other contexts, discovery may arise from mineral exploration, such as Bolivia's realization in the 1980s that it held the world's largest reserves of lithium, or experimentation with soil conditions, which in the 1970s allowed Brazil to begin exporting soybeans (Bustos et al. 2015). Seemingly random discoveries are often followed by equally unexpected declines in global standing. While Brazil remains a leading exporter of soybeans, the rise of smart phones has dented Finland's prominence in mobile technology, Intel's decision to close its operations in Costa Rica is abruptly shifting the country's comparative advantage, and ongoing conflicts over property rights have limited Bolivia's exports of lithium. The parsimonious stochastic process that we specify treats discovery as random and the erosion of these discoveries as governed by reversion to the mean.

In Section 2 we present a theoretical motivation for our gravity specification. In Section 3 we describe the data and gravity model estimates, and document stationarity and heavy tails in export advantage as well as churning in top export goods. In Section 4 we introduce a stochastic process that generates a cross-sectional distribution consistent with heavy tails and embeds innovations consistent with churning, and we derive a GMM estimator for this process. In Section 5 we present estimates and evaluate the fit of the model. In Section 6 we conclude.

2 Theoretical Motivation

We use the EK model to motivate our definitions of export capability and absolute advantage, and describe our approach for extracting these measures from the gravity equation of trade.

2.1 Export capability, absolute advantage, and comparative advantage

In EK, an industry consists of many product varieties. The productivity q of a source-country s firm that manufactures a variety in industry i is determined by a random draw from a Fréchet distribution with CDF $F_Q(q) = \exp\{-(q/\underline{q}_{is})^{-\theta}\}$ for $q > 0$. The location parameter $\underline{q}_{is}$ determines the typical productivity level of a firm in the industry while the shape parameter θ controls the dispersion in productivity across firms. Consumers, who have CES preferences over product varieties within an industry, buy from the firm that delivers a variety at the lowest price. With marginal-cost pricing, a higher productivity draw makes a firm more likely to be the lowest-price supplier of a variety to a given market.

Comparative advantage stems from the location of the industry productivity distribution, given by $\underline{q}_{is}$, which may vary by country and industry. In a country-industry with a higher $\underline{q}_{is}$, firms are more likely to have a high productivity draw, such that in this country-industry a larger fraction of firms succeeds in exporting to multiple destinations. Consider the many-industry version of the EK model in Costinot et al. (2012). Exports by source country s to destination country d in industry i can be written as,

$$X_{isd} = \frac{\left(w_s \tau_{isd}/\underline{q}_{is}\right)^{-\theta}}{\sum_{\varsigma} \left(w_{\varsigma} \tau_{i\varsigma d}/\underline{q}_{i\varsigma}\right)^{-\theta}} \mu_i Y_d, \tag{1}$$

where w_s is the unit production cost in source country s, τ_{isd} is the iceberg trade cost between s and d in industry i, μ_i is the Cobb-Douglas share of industry i in global expenditure, and Y_d is national expenditure in country d. Taking logs of (1), we obtain a gravity equation for bilateral trade

$$\ln X_{isd} = k_{is} + m_{id} - \theta \ln \tau_{isd}, \tag{2}$$

where $k_{is} \equiv \theta \ln(\underline{q}_{is}/w_s)$ is source country s's log *export capability* in industry i, which is a function of the country-industry's efficiency ($\underline{q}_{is}$) and the country's unit production cost (w_s),[7] and

$$m_{id} \equiv \ln \left[\mu_i Y_d / \sum_{\varsigma} \left(w_{\varsigma} \tau_{i\varsigma d} / \underline{q}_{i\varsigma} \right)^{-\theta} \right]$$

is the log of effective import demand by country d in industry i, which depends on national expenditure on goods in the industry divided by an index of the toughness of industry competition in the country.

Though we focus on EK, any trade model that has a gravity structure will generate exporter-industry fixed effects and a reduced-form expression for export capability (k_{is}). In the Armington (1969) model, as applied by Anderson and van Wincoop (2003), export capability is a country's endowment of a good relative to its remoteness from the rest of the world. In Krugman (1980), export capability equals the number of varieties a country produces in an industry times effective industry marginal production costs. In Melitz (2003), export capability is analogous to that in Krugman adjusted by the Pareto lower bound for productivity in the industry. In a Heckscher-Ohlin model (Deardorff 1998), export capability reflects the relative size of a country's industry based on factor endowments and sectoral factor intensities. The common feature of these models is that export capability is related to a country's productive potential in an industry, be it associated with resource supplies, a home-market effect, or the distribution of firm-level productivity.

Looking forward to the estimation, the presence of the importer-industry fixed effect m_{id} in (2) implies that export capability k_{is} is only identified up to an industry normalization. We therefore re-express export capability as the deviation from its global industry mean $(1/S) \sum_{\varsigma=1}^{S} k_{i\varsigma}$, where S is the number of source countries. Exponentiating this value, we measure *absolute advantage* of source country s in industry i as

$$A_{is} \equiv \frac{\exp\{k_{is}\}}{\exp\left\{\frac{1}{S}\sum_{\varsigma=1}^{S} k_{i\varsigma}\right\}} = \frac{(\underline{q}_{is}/w_s)^{\theta}}{\exp\left\{\frac{1}{S}\sum_{\varsigma=1}^{S} (\underline{q}_{i\varsigma}/w_{\varsigma})^{\theta}\right\}}. \tag{3}$$

The normalization in (3) differences out both worldwide industry supply conditions, such as shocks to global TFP, and worldwide industry demand conditions, such as variation in the expenditure share μ_i.

Our measure of absolute advantage is one of several possible starting points as we work towards comparative advantage. When A_{is} rises for country-industry is, we say that country s's absolute advantage has increased in industry i even though it is only strictly the case that its export capability has risen relative to the global geometric mean for i. In fact, the country's export capability in i may have gone up relative to some countries and fallen

[7]Export capability k_{is} depends on endogenously determined production costs w_s and therefore is not a primitive. The EK model does not yield a closed-form solution for wages, so we cannot solve for export capabilities as explicit functions of the $\underline{q}_{is}$'s. In a model with labor as the single primary factor of production, the $\underline{q}_{is}$'s are the only country and industry-specific fundamentals—other than trade costs—that determine factor prices, implying in turn that the w_s's are implicit functions of the $\underline{q}_{is}$'s.

relative to others. Our motivation for using the deviation from the industry geometric mean to define absolute advantage is that this definition simplifies the specification of a stochastic process for export capability. Rather than specifying export capability itself, we model its deviation from a worldwide industry trend, which frees us from having to model the global trend component.

To relate our use of absolute advantage A_{is} to conventional approaches, average (2) over destinations and define (harmonic) log exports from source country s in industry i at the country's industry trade costs as

$$\ln \bar{X}_{is} \equiv k_{is} + \frac{1}{D}\sum_{d=1}^{D} m_{id} - \frac{1}{D}\sum_{d=1}^{D} \theta \ln \tau_{isd}, \tag{4}$$

where D is the number of destination markets. We say that country s has a comparative advantage over country ς in industry i relative to industry j if the following familiar condition holds:

$$\frac{\bar{X}_{is}/\bar{X}_{i\varsigma}}{\bar{X}_{js}/\bar{X}_{j\varsigma}} = \frac{A_{is}/A_{i\varsigma}}{A_{js}/A_{j\varsigma}} > 1. \tag{5}$$

Intuitively, absolute advantage defines country relative exports, once we neutralize the distorting effects of trade costs and proximity to market demand on trade flows, as in (4). In practice, a large number of industries and countries makes it cumbersome to conduct double comparisons of country-industry is to all other industries and all other countries, as suggested by (5). The definition in (3) simplifies this comparison in the *within-industry dimension* by setting the "comparison country" in industry i to be the global mean across countries in i. In the final estimation strategy that we develop in Section 4, we will further normalize the comparison in the *within-country dimension* by estimating the absolute advantage of the "comparison industry" for country s, consistent with an arbitrary stochastic country-wide growth process. Demeaning in the industry dimension and then estimating the most suitable normalization in the country dimension makes our empirical approach consistent with both worldwide stochastic industry growth and stochastic national country growth.

Our concept of export capability k_{is} can be related to the deeper origins of comparative advantage by treating the country-industry specific location parameter $\underline{q}_{is}$ as the outcome of an exploration and innovation process. In Eaton and Kortum (1999, 2010), firms generate new ideas for how to produce existing varieties more efficiently. The efficiency q of a new idea is drawn from a Pareto distribution with CDF $G(q) = (q/\underline{x}_{is})^{-\theta}$, where $\underline{x}_{is} > 0$ is the minimum efficiency. New ideas arrive in continuous time according to a Poisson process, with intensity rate $\rho_{is}(t)$. At date t, the number of ideas with at least efficiency q is then distributed Poisson with parameter $T_{is}(t)\, q^{-\theta}$, where $T_{is}(t)$ is the number of previously discovered ideas that are available to producers and that is in turn a function of $\underline{x}_{is}^{\theta}$ and past realizations of $\rho_{is}(t)$.[8] Setting $T_{is}(t) = \underline{q}_{is}(t)^{\theta}$, this framework yields

[8] Eaton and Kortum (2010) allow costly research effort to affect the Poisson intensity rate and assume that there is "no forgetting" such

identical predictions for the volume of bilateral trade as in equation (1). Our empirical approach is to treat the stock of ideas available to a country in an industry $T_{is}(t)$—relative to the global industry mean stock of ideas $(1/S)\sum_{\varsigma=1}^{S} T_{i\varsigma}(t)$—as following a stochastic process.[9]

2.2 Estimating the gravity model

Allowing for measurement error in trade data or unobserved trade costs, we can introduce a disturbance term into the gravity equation (2), converting it into a linear regression model. With data on bilateral industry trade flows for many importers and exporters, we can obtain estimates of the exporter-industry and importer-industry fixed effects from an OLS regression. The gravity model that we estimate is

$$\ln X_{isdt} = k_{ist} + m_{idt} + \mathbf{r}'_{sdt}\mathbf{b}_{it} + v_{isdt}, \tag{6}$$

where we have added a time subscript t. We include dummy variables to measure exporter-industry-year k_{ist} and importer-industry-year m_{idt} terms. The regressors $\mathbf{r}_{sdt}$ represent the determinants of bilateral trade costs, and v_{isdt} is a residual that is mean independent of $\mathbf{r}_{sdt}$. The variables we use to measure trade costs $\mathbf{r}_{sdt}$ in (6) are standard gravity covariates, which do not vary by industry.[10] However, we do allow the coefficient vector $\mathbf{b}_{it}$ on these variables to differ by industry and by year.[11] Absent annual measures of industry-specific trade costs for the full sample period, we model these costs via the interaction of country-level gravity variables and time-and-industry-varying coefficients.

The values that we will use for empirical analysis are the deviations of the estimated exporter-industry-year dummies from global industry means. The empirical counterpart to the definition of absolute advantage in (3) for source country s in industry i is

$$A_{ist} = \frac{\exp\left\{k_{ist}^{\text{OLS}}\right\}}{\exp\left\{\frac{1}{S}\sum_{\varsigma=1}^{S} k_{i\varsigma t}^{\text{OLS}}\right\}} = \frac{\exp\left\{k_{ist}\right\}}{\exp\left\{\frac{1}{S}\sum_{\varsigma=1}^{S} k_{i\varsigma t}\right\}}, \tag{7}$$

that all previously discovered ideas are available to firms. In our simple sketch, we abstract away from research effort and treat the stock of knowledge available to firms in a country (relative to the mean across countries) as stochastic.

[9] Buera and Oberfield (2016) microfound the innovation process in Eaton and Kortum (2010) by allowing agents to transmit ideas within and across borders through trade. A Fréchet distribution for country-industry productivity emerges as an equilibrium outcome in this environment, where the location parameter of this distribution reflects the current stock of ideas in a country.

[10] These include log distance between the importer and exporter, the time difference (and time difference squared) between the importer and exporter, a contiguity dummy, a regional trade agreement dummy, a dummy for both countries being members of GATT, a common official language dummy, a common prevalent language dummy, a colonial relationship dummy, a common empire dummy, a common legal origin dummy, and a common currency dummy.

[11] We estimate (6) separately by industry and by year. Since in each year the regressors are the same across industries for each bilateral exporter-importer pair, there is no gain to pooling data across industries in the estimation, which helps reduce the number of parameters to be estimated in each regression.

where k_{ist}^{OLS} is the OLS estimate of k_{ist} in (6).

As is well known (Silva and Tenreyro 2006, Head and Mayer 2014), the linear regression model (6) is inconsistent with the presence of zero trade flows, which are common in bilateral data. We recast EK to allow for zero trade by following Eaton et al. (2012), who posit that in each industry in each country only a finite number of firms make productivity draws, meaning that in any realization of the data there may be no firms from country s that have sufficiently high productivity to profitably supply destination market d in industry i. Instead of augmenting the expected log trade flow $\mathbb{E}[\ln X_{isd}]$ from gravity equation (2) with a disturbance, Eaton et al. (2012) consider the expected share of country s in the market for industry i in country d, $\mathbb{E}[X_{isd}/X_{id}]$, and write this share in terms of a multinomial logit model. This approach requires that one know total expenditure in the destination market, X_{id}, including a country's spending on its own goods. Since total spending is unobserved in our data, we invoke independence of irrelevant alternatives and specify the dependent variable as the expectation for the share of source country s in import purchases by destination d in industry i:

$$\mathbb{E}\left[\frac{X_{isdt}}{\sum_{\varsigma\neq d} X_{i\varsigma dt}}\right] = \frac{\exp\left\{k_{ist} - \mathbf{r}'_{sdt}\mathbf{b}_{it}\right\}}{\sum_{\varsigma\neq d}\exp\left\{k_{i\varsigma t} - \mathbf{r}'_{\varsigma dt}\mathbf{b}_{it}\right\}}. \tag{8}$$

In practice, estimation of (8) turns out to be well approximated by estimation of the Poisson pseudo-maximum-likelihood (PPML) gravity model proposed by Silva and Tenreyro (2006). We re-estimate exporter-industry-year fixed effects by applying PPML to (8).[12]

Our baseline measure of absolute advantage relies on regression-based estimates of exporter-industry-year fixed effects. Estimates of these fixed effects may become imprecise when a country exports a good to only a few destinations in a given year. As an alternative measure of export performance, we use the Balassa (1965) measure of revealed comparative advantage:

$$RCA_{ist} \equiv \frac{\sum_d X_{isdt}/\sum_\varsigma \sum_d X_{i\varsigma dt}}{\sum_j \sum_d X_{jsdt}/\sum_j \sum_\varsigma \sum_d X_{j\varsigma dt}}. \tag{9}$$

While the RCA index is ad hoc and does not correct for distortions in trade flows introduced by trade costs or proximity to market demand, it has the appealing attribute of being based solely on raw trade data. Throughout our analysis we will employ OLS and PPML gravity-based measures of absolute advantage (7) alongside the Balassa RCA measure (9). Reassuringly, our results for the three measures are quite similar.

[12] We thank Sebastian Sotelo for estimation code.

3 Data and Main Regularities

The data for our analysis are World Trade Flows from Feenstra et al. (2005), which are based on SITC revision 1 industries for 1962 to 1983 and SITC revision 2 industries for 1984 to 2007. We create a consistent set of country aggregates in these data by maintaining as single units countries that divide or unite over the sample period.[13] To further maintain consistency in the countries present, we restrict the sample to nations that trade in all years and that exceed a minimal size threshold, which leaves 116 country units.[14] The switch from SITC revision 1 to revision 2 in 1984 led to the creation of many new industry categories. To maintain a consistent set of SITC industries over the sample period, we aggregate industries to a combination of two- and three-digit categories.[15] These aggregations and restrictions leave 133 industries in the data. In an extension of our main analysis, we limit the sample to SITC revision 2 data for 1984 forward, so we can check the sensitivity of our results to industry aggregation by using two-digit (60 industries) and three-digit definitions (225 industries), which bracket the industry definitions that we use for the full-sample period.[16]

A further set of country restrictions is required to estimate importer and exporter fixed effects. For coefficients on exporter-industry dummies to be comparable over time, it is important to require that destination countries import a product in all years. Imposing this restriction limits the sample to 46 importers, which account for an average of 92.5% of trade among the 116 country units. In addition, we need that exporters ship to overlapping groups of importing countries. As Abowd et al. (2002) show, such connectedness assures that all exporter fixed effects are separately identified from importer fixed effects. This restriction leaves 90 exporters in the sample that account for an average of 99.4% of trade among the 116 country units. Using our sample of 90 exporters, 46 importers, and 133 industries, we estimate the gravity equation (6) separately by industry i and year t and then extract absolute advantage A_{ist} given by (7). Data on gravity variables are from CEPII.org.

[13] These are the Czech Republic, the Russian Federation, and Yugoslavia. We join East and West Germany, Belgium and Luxembourg, as well as North and South Yemen.

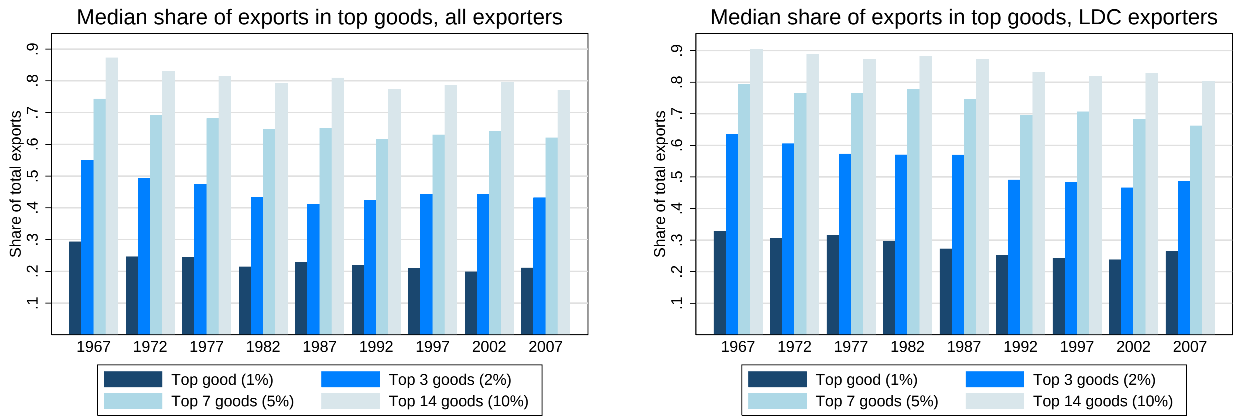
[14] This reporting restriction leaves 141 importers (97.7% of world trade) and 139 exporters (98.2% of world trade) and is roughly equivalent to dropping small countries from the sample. For consistency in terms of country size, we drop countries with fewer than 1 million inhabitants in 1985, reducing the sample to 116 countries (97.4% of world trade).

[15] There are 226 three-digit SITC industries that appear in all years, which account for 97.6% of trade in 1962 and 93.7% in 2007. Some three-digit industries frequently have their trade reported only at the two-digit level (which accounts for the just reported decline in trade shares for three-digit industries). We aggregate over these industries, creating 143 industry categories that are a mix of SITC two and three-digit sectors. From this group we drop non-standard industries: postal packages (SITC 911), special transactions (SITC 931), zoo animals and pets (SITC 941), non-monetary coins (SITC 961), and gold bars (SITC 971). We further exclude uranium (SITC 286), coal (SITC 32), petroleum (SITC 33), natural gas (SITC 341), and electrical current (SITC 351), which violate the Abowd et al. (2002) requirement of connectedness for estimating identified exporter fixed effects in many years.

[16] In an earlier version of our paper, we estimated OLS gravity equations for four-digit SITC revision 2 products (682 industries). PPML estimates at the four-digit level turn out to be quite noisy, owing to the many exporters in industries at this level of disaggregation that ship goods to no more than a few importers. Consequently, we exclude data on four-digit industries from the analysis.

Figure 1: **Concentration of Exports**

(1a) All exporters **(1b) LDC exporters**



Source: WTF (Feenstra et al. 2005, updated through 2008) for 133 time-consistent industries in 90 countries from 1962-2007.
Note: Shares of industry i's export value in country s's total export value: $X_{ist}/(\sum_j X_{jst})$. For the classification of less developed countries (LDC) see the Supplementary Material (Section S.1).

3.1 Stable heavy tails in export advantage

We first characterize export behavior across industries for each country. For an initial take on the concentration of exports in leading products, we tabulate the share of industry exports in a country's total exports across the 133 industries $X_{ist}/(\sum_j X_{jst})$ and then average these shares across the current and preceding two years.

In **Figure 1a**, we display median export shares across the 90 countries in our sample for the top export industry as well as the top 3, top 7, and top 14 industries, which correspond to the top 1%, 2%, 5% and 10% of products. For the typical country, a handful of industries dominate exports. The median export share of the top export good is 24.6% in 1972, which declines modestly to 21.4% in 1982 and then remains stable around this level for the next two-and-a-half decades. For the top 3 products, the median export share declines slightly from the 1960s to the 1970s and then is stable from the early 1980s onward, averaging 43.5% for 1982 to 2007. The median export shares of the top 7 and top 14 products display a similar pattern, averaging 63.1% and 78.6%, respectively, for 1982 to 2007. **Figure 1b**, which limits the sample to less developed countries, reveals similar patterns, though median export shares of top products are somewhat higher.[17]

An obvious concern about using export shares to measure export concentration is that these values may be distorted by demand conditions. Exports in some industries may be large simply because these industries capture a relatively large share of global expenditure, leading the same industries, such as automobiles or electronics, to

[17] See the Supplementary Material (Section S.1) for the set of countries. In analyses of developing-country trade, Easterly and Reshef (2010) document the tendency of a small number of destination markets to dominate national exports by industry and Freund and Pierola (2013) describe the prominent export role of a country's largest firms.

be the top exporter in many countries. Similarly, a country's geographic proximity to major consumer markets may contribute to its apparent export success beyond its inherit capability. To control for variation in industry size and geographic proximity that affect trade volumes beyond a country-industry's export capability, we turn to our measure of absolute advantage in (7) expressed in logs as $\ln A_{ist}$.[18]

Figure 2 depicts the full distribution of absolute advantage across industries for 12 countries in 2007. Here, we plot the log number of industries for exporter s that have at least a given level of absolute advantage in year t against the corresponding log level of industry absolute advantage $\ln A_{ist}$. By design, the plot characterizes the cumulative distribution of absolute advantage by country and by year (Axtell 2001, Luttmer 2007). Plots for 28 countries in 1967, 1987 and 2007 are shown in Appendix **Figures A1, A2** and **A3**. While the lower cutoff for absolute advantage shifts right over time, the shape of the cross sectional CDF is remarkably stable across countries and years. This shape stability of the cross-sectional absolute advantage distribution suggests that comparative advantage is trend stationary, a robust feature that we will revisit under varying perspectives.

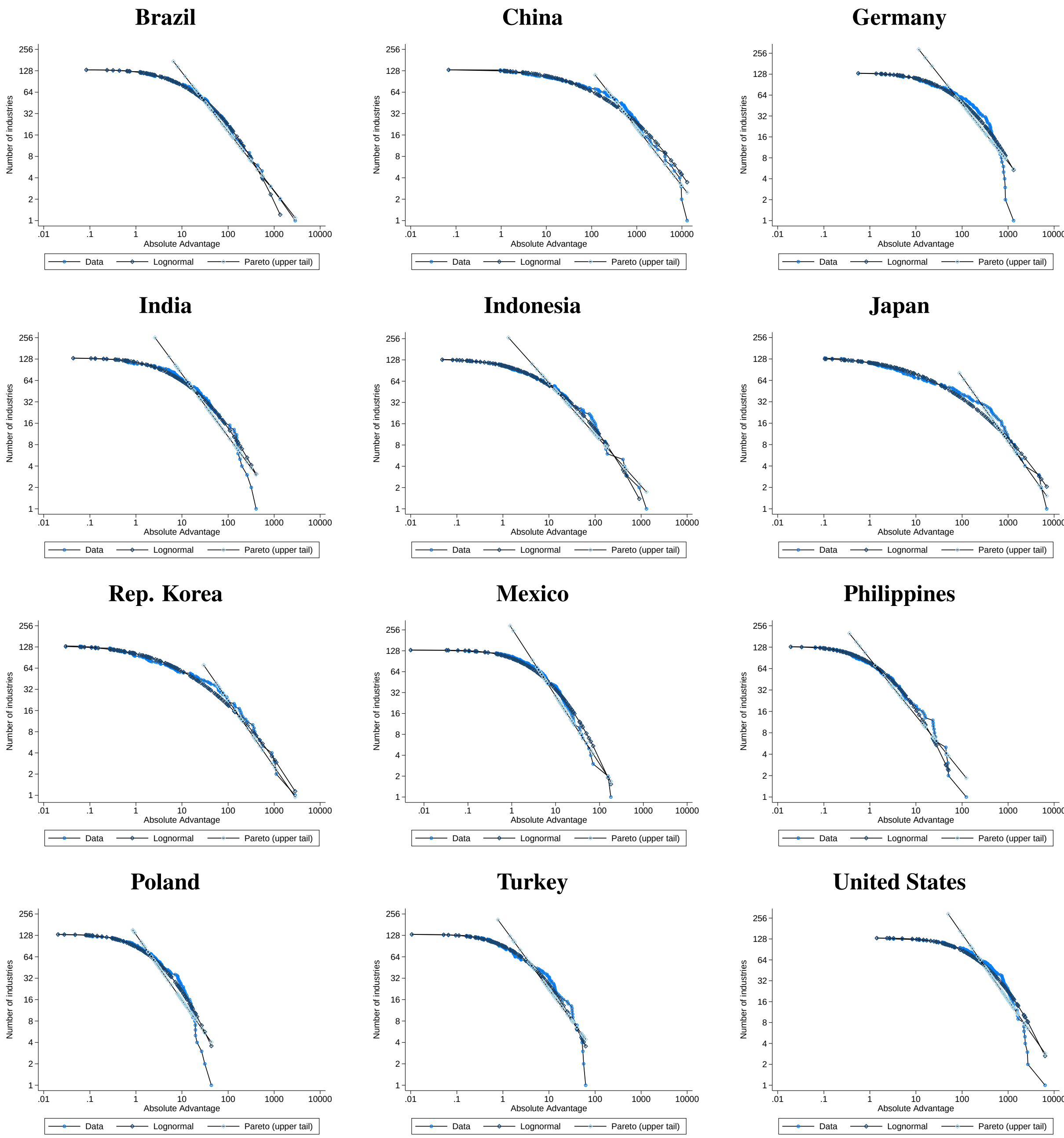
The figures also graph the fit of absolute advantage to a Pareto distribution and to a log normal distribution using maximum likelihood, where each distribution is fit separately for each country in each year. The Pareto and the log normal are common choices in the literatures on the distribution of city and firm sizes (Sutton 1997). For the Pareto distribution, the cumulative distribution plot is linear in the logs, whereas the log normal distribution generates a relationship that is concave to the origin.

The cumulative distribution plots clarify that the empirical distribution of absolute advantage is not Pareto. The log normal, in contrast, fits the data closely. The concavity of the cumulative distribution plots drawn for the data indicate that gains in absolute advantage fall off progressively more rapidly as one moves up the rank order of absolute advantage, a feature absent from the scale-invariant Pareto but characteristic of the log normal. Consistent with **Figure 1**, the upper tails of the distribution are heavy. Across all countries and years, the ratio of the mean to the median is 11.1 for absolute advantage based on our baseline OLS estimates of export capability, 23.5 for absolute advantage based on PPML estimates, and 1.2 for the Balassa RCA index, which further standardizes absolute advantage to comparative advantage.[19] Though the log normal approximates the shape of the distribution for absolute advantage well, there are certain discrepancies between the fitted log normal

[18] In the Supplementary Material (Table S1), we show the top two products in terms of $\ln A_{ist}$ for select countries and years. To remove the effect of national market size and make values comparable across countries, we normalize log absolute advantage by its country mean, which produces a double log difference—a country-industry's log deviation from the global industry mean less the country-wide average across all industries—and captures comparative advantage. The magnitudes of export advantage are enormous. In 2007, comparative advantage in the top product is over 300 log points in 88 of the 90 exporting countries. To verify that our measure of export advantage does not peg obscure industries as top sectors, in the Supplementary Material (Figure S1) we plot $\ln A_{ist}$ against the log of the share of the industry in national exports $\ln(X_{ist}/(\sum_j X_{jst}))$. In all years, there is a strongly positive correlation between log absolute advantage and the log industry share of national exports (0.77 in 1967, 0.78 in 1987, and 0.83 in 2007).

[19] To compute the reported mean-median ratios, we omit outliers consistent with later estimation and weight by sector counts within country-years.

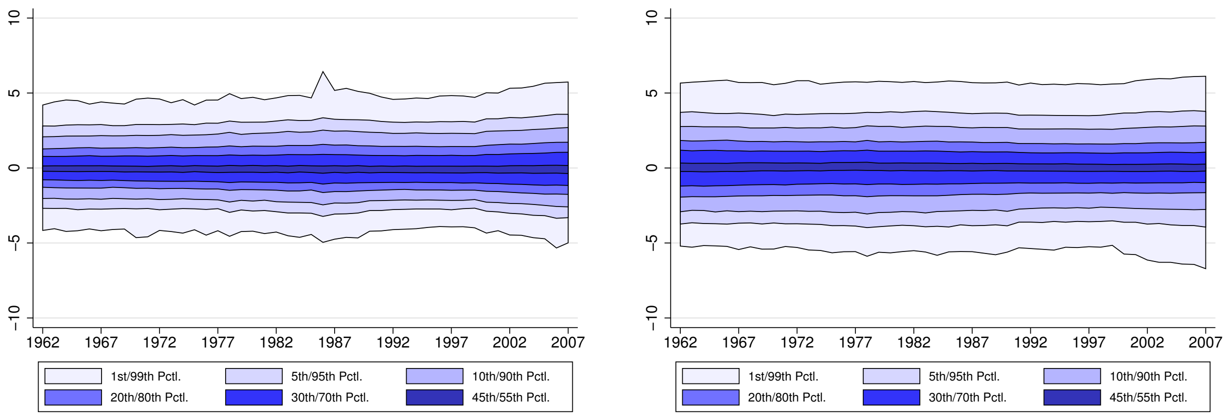
Figure 2: **Cumulative Probability Distribution of Absolute Advantage for Select Countries in 2007**



Source: WTF (Feenstra et al. 2005, updated through 2008) for 133 time-consistent industries in 90 countries in 2005-2007 and CEPII.org; three-year means of OLS gravity measures of export capability (log absolute advantage) $k = \ln A$ from (6).
Note: The graphs show the frequency of industries (the cumulative probability $1 - F_A(a)$ times the total number of industries $I = 133$) on the vertical axis plotted against the level of absolute advantage a (such that $A_{ist} \geq a$) on the horizontal axis. Both axes have a log scale. The fitted Pareto and log normal distributions are based on maximum likelihood estimation by country s in year $t = 2007$ (Pareto fit to upper five percentiles only).

Figure 3: **Percentiles of Comparative Advantage Distributions by Year**

(3a) OLS gravity measures **(3b) Balassa index**



Source: WTF (Feenstra et al. 2005, updated through 2008) for 133 time-consistent industries in 90 countries from 1962-2007; OLS gravity measures of export capability (log absolute advantage) $k = \ln A$ from (6).
Note: We obtain log comparative advantage as the residuals from OLS projections on industry-year and source country-year effects (δ_{it} and δ_{st}) for (a) OLS gravity measures of log absolute advantage $\ln A_{ist}$ and (b) the log Balassa index of revealed comparative advantage $\ln RCA_{ist} = \ln(X_{ist}/\sum_{\varsigma} X_{i\varsigma t})/(\sum_j X_{jst}/\sum_j \sum_{\varsigma} X_{j\varsigma t})$.

plots and the raw data plots. For some countries, the number of industries in the upper tail drops too fast (is more concave), relative to what the log normal distribution predicts. These discrepancies motivate our specification of a generalized logistic diffusion for absolute advantage in Section 4.

To verify that our findings are not the byproduct of failing to control for zero bilateral trade in the gravity estimation, we also show plots based on PPML estimates of export capability, with similar results. To verify that the graphed cross-section distributions are not a byproduct of specification error in estimating the gravity model, we repeat the plots using the Balassa RCA index in 1987 and 2007, again with similar results. And to verify that the patterns we uncover are not a consequence of arbitrary industry aggregation, we construct plots at the three-digit level based on SITC revision 2 data in 1987 and 2007, yet again with similar results.[20]

Figures A1, A2 and **A3** in the Appendix provide visual evidence that the heavy tails of the distribution of absolute advantage for individual countries are stable over time. To substantiate this property of the data, we pool industry-level measures of comparative advantage across countries and plot the percentiles of this global distribution in each year, as shown in **Figure 3** for OLS-based measures of export capability and for Balassa RCA indexes.[21] The plots for the 5th/95th, 20th/80th, 30th/70th, and 45th/55th percentiles are, with minor fluctuation, parallel to the horizontal axis. This is a strong indication that the global distribution of comparative

[20]Each of these additional sets of results is available in the Supplementary Material: Figures S2 and S3 for the PPML estimates, Figures S4 and S5 for the Balassa measure, and Figures S6, S7, S8 and S9 for the two- and three-digit industry definitions under SITC revision 2.

[21]The Supplementary Material (Figure S10) shows percentile plots for PPML-based measures of export capability.

advantage is stationary. If it were the case that comparative advantage was degenerate, the percentile lines would slope downward from above the mean and upward from below the mean, as the distribution became increasingly compressed over time, a pattern clearly not in evidence. If, instead, the distribution of comparative advantage was non-stationary, we would see the upper percentile lines drifting upward and the lower percentile lines drifting downward. There is mild drift only in the extreme tails of the distribution, the 1^{st} and 99^{th} percentiles, and there only during the early 2000s, a pattern which stalls or reverses after 2005.

Before examining the time series of export advantage in more detail, we consider whether a log normal distribution of absolute advantage could be an incidental consequence of the gravity estimation. The exporter-industry fixed effects are estimated sample parameters, which by the Central Limit Theorem converge to being normally distributed around their respective population parameters as the sample size becomes large. However, normality of this log export capability estimator does not imply that the cross-sectional distribution of absolute advantage becomes log normal. If no other element but the residual noise from gravity estimation generated log normality in absolute advantage, then the cross-sectional distribution of absolute advantage between industries in a country would be degenerate around a single mean. The data are clearly in favor of non-degeneracy for the distribution of absolute advantage. **Figure 2** and its counterparts (**Figures A1, A2** and **A3** in the Appendix) document that industries within a country differ markedly in terms of their mean export capability. The distribution of Balassa revealed comparative advantage is also approximately log normal, which indicates that non-regression based measures of comparative advantage elicit similar distributional patterns.

3.2 Churning in export advantage

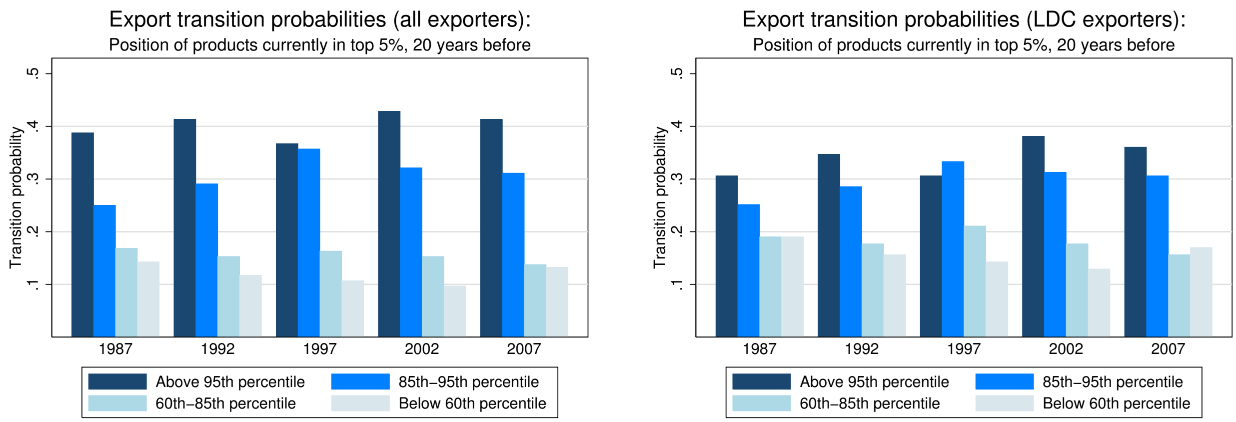
The stable distribution plots of absolute advantage give an impression of little variability. The strong concavity in the cross-sectional plots is present in all countries and in all years. Yet, this cross-sectional stability masks considerable turnover in industry rankings of absolute advantage behind the cross-sectional distribution. Of the 90 total exporters, 68 have a change in the top comparative-advantage industry between 1987 and 2007.[22] Over this period, Canada's top good switches from sulfur to wheat, China's from fireworks to telecommunications equipment, Egypt's from cotton to crude fertilizers, India's from tea to precious stones, and Poland's from barley to furniture. Moreover, most new top products in 2007 were not the number one or two product in 1987 but came from lower down the ranking. Churning thus appears to be both pervasive and disruptive.

To characterize turnover in industry export advantage, in **Figure 4** we calculate the fraction of top products in a given year that were also top products in the past. For each country in each year, we identify where in the distribution the top 5% of absolute-advantage products (in terms of A_{ist}) were 20 years before, with the categories

[22] Evidence of this churning is seen in the Supplementary Material (Table S1).

Figure 4: **Absolute Advantage Transition Probabilities**

(4a) All exporters | **(4b) LDC exporters**



Source: WTF (Feenstra et al. 2005, updated through 2008) for 133 time-consistent industries in 90 countries from 1962-2007; OLS gravity measures of export capability (log absolute advantage) $k = \ln A$ from (6).
Note: The graphs show the percentiles of products is that are currently among the top 5% of products, 20 years earlier. The sample is restricted to products (country-industries) is with current absolute advantage A_{ist} in the top five percentiles ($1 - F_A(A_{ist}) \geq .05$), and then grouped by frequencies of percentiles twenty years prior, where the past percentile is $1 - F_A(A_{is,t-20})$ of the same product (country-industry) is. For the classification of less developed countries (LDC) see the Supplementary Material (Section S.1).

being top 5%, next 10%, next 25% or bottom 60%. We then average across outcomes for the 90 export countries. The fraction of top 5% products in a given year that were also top 5% products two decades earlier ranges from a high of 42.9% in 2002 to a low of 36.7% in 1997. Averaging over all years, the share is 40.2%, indicating a 60% chance that a good in the top 5% in terms of absolute advantage today was not in the top 5% two decades before. On average, 30.6% of new top products come from the 85th to 95th percentiles, 15.5% come from the 60th to 85th percentiles, and 11.9% come from the bottom six deciles. Outcomes are similar when we limit the sample to developing economies.

Turnover in top export goods suggests that over time export advantage dissipates—countries' strong sectors weaken and their weak sectors strengthen—as would be consistent with mean reversion. We test for mean reversion in export capability by specifying the following AR(1) process,

$$k^{\text{OLS}}_{is,t+10} - k^{\text{OLS}}_{ist} = \rho\, k^{\text{OLS}}_{ist} + \delta_{it} + \delta_{st} + \varepsilon_{is,t+10}, \tag{10}$$

where k^{OLS}_{ist} is the OLS estimate of log export capability from gravity equation (6). In (10), the dependent variable is the ten-year change in export capability and the predictors are the initial value of export capability and dummies for the industry-year δ_{it} and for the country-year δ_{st}. We choose a long time difference for export capability—a full decade—to help isolate systematic variation in country export advantages. Controlling for industry-year fixed effects converts export capability into a measure of absolute advantage; controlling further for country-year fixed effects allows us to evaluate the dynamics of comparative advantage. The coefficient ρ captures the fraction of comparative advantage that decays over ten years. The specification in (10) is similar to the productivity convergence regressions reported in Levchenko and Zhang (2013), except that we use trade data to calculate country advantage in an industry, examine industries at a considerably more disaggregate level, and include both manufacturing and nonmanufacturing sectors in the analysis. Because we estimate log export capability k_{ist}^{OLS} from the first-stage gravity estimation in (6), we need to correct the standard errors in (10) for the presence of generated variables. To do so, we apply a generated-variable correction discussed in Appendix D.[23]

Table 1 presents coefficient estimates for equation (10). The first three columns report results for log export capability based on OLS, the next three for log export capability based on PPML, and the final three for the log Balassa RCA index. Estimates for ρ are uniformly negative and precisely estimated, consistent with mean reversion in export advantage. We soundly reject the hypothesis that there is no decay (H_0: $\rho = 0$) and also the hypothesis that there is instantaneous dissipation (H_0: $\rho = -1$). Estimates for the full sample of countries and industries in columns 1, 4, and 7 are similar in value, equal to -0.35 when using OLS log export capability, -0.32 when using PPML log export capability, and -0.30 when using log RCA. These magnitudes indicate that over the period of a decade the typical country-industry sees approximately one-third of its comparative advantage (or disadvantage) erode. In columns 2, 5, and 8, we present comparable results for the subsample of developing countries. Decay rates for this group are larger than the worldwide averages in columns 1, 4, and 7, indicating that in less-developed economies mean reversion in comparative advantage is more rapid. In columns 3, 6, and 9, we present results for nonmanufacturing industries (agriculture, mining, and other primary commodities). For PPML export capability and Balassa RCA, decay rates for the nonmanufacturing sector are similar to those for the full sample of industries.

As an additional robustness check, we re-estimate (10) for the period 1984-2007 using data from the SITC revision 2 sample, reported in Appendix **Table A1**. Estimated decay rates are comparable to those in **Table 1**. At either the two-digit level (60 industries) or three-digit level (224 industries), the decay-rate estimates based on PPML export capability and RCA indexes are similar to those for the baseline combined two- and three-digit level (133 industries), with estimates based on OLS export capability being somewhat more variable. Because these additional samples use data for the 1984-2007 period and the original sample uses the full 1962-2007 period, these results also serve as a robustness check on the stability in coefficient estimates over time.

Our finding that decay rates imply incomplete mean reversion is further evidence against absolute advantage being incidental. Suppose that the cumulative distribution plots of log absolute advantage reflected random varia-

[23] This correction is for GMM. For a discussion of the OLS correction as a special case of the GMM correction, see the Supplementary Material (Section S.2).

Table 1: OLS ESTIMATES OF COMPARATIVE ADVANTAGE DECAY, 10-YEAR TRANSITIONS

	OLS gravity k			PPML gravity k			$\ln RCA$		
	All	LDC	Nonmanf.	All	LDC	Nonmanf.	All	LDC	Nonmanf.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Decay Regression Coefficients									
Decay rate ρ	-0.349	-0.454	-0.450	-0.320	-0.358	-0.322	-0.303	-0.342	-0.293
	(0.002)***	(0.002)***	(0.003)***	(0.0002)***	(0.0003)***	(0.0003)***	(0.01)***	(0.013)***	(0.012)***
Var. of residual s^2	2.089	2.408	2.495	2.709	3.278	3.123	2.318	2.849	2.561
	(0.024)***	(0.026)***	(0.042)***	(0.013)***	(0.018)***	(0.021)***	(0.006)***	(0.009)***	(0.009)***
Implied Ornstein-Uhlenbeck (OU) Parameters									
Dissipation rate η	0.276	0.292	0.280	0.198	0.179	0.173	0.222	0.199	0.195
	(0.003)***	(0.003)***	(0.005)***	(0.0009)***	(0.001)***	(0.001)***	(0.006)***	(0.006)***	(0.006)***
Intensity of innovations σ	0.558	0.644	0.654	0.623	0.703	0.670	0.570	0.648	0.596
	(0.003)***	(0.004)***	(0.006)***	(0.001)***	(0.002)***	(0.002)***	(0.005)***	(0.009)***	(0.006)***
Observations	324,978	202,010	153,768	320,310	199,724	149,503	324,983	202,014	153,773
Adjusted R^2 (within)	0.222	0.267	0.262	0.282	0.290	0.266	0.216	0.224	0.214
Years t	36	36	36	36	36	36	36	36	36
Industries i	133	133	68	133	133	68	133	133	68
Source countries s	90	62	90	90	62	90	90	62	90

Source: WTF (Feenstra et al. 2005, updated through 2008) for 133 time-consistent industries in 90 countries from 1962-2007 and CEPII.org; OLS and PPML gravity measures of export capability (log absolute advantage) $k = \ln A$ from (8).
Note: Reported figures for ten-year changes. Variables are OLS and PPML gravity measures of log absolute advantage $\ln A_{ist}$ and the log Balassa index of revealed comparative advantage $\ln RCA_{ist} = \ln(X_{ist}/\sum_{\varsigma} X_{i\varsigma t})/(\sum_j X_{jst}/\sum_j \sum_{\varsigma} X_{j\varsigma t})$. OLS estimation of the ten-year decay rate ρ from

$$k_{is,t+10} - k_{ist} = \rho\, k_{ist} + \delta_{it} + \delta_{st} + \epsilon_{is,t+10},$$

conditional on industry-year and source country-year effects δ_{it} and δ_{st} for the full pooled sample (column 1-2) and subsamples (columns 3-6). The implied dissipation rate η and squared innovation intensity σ^2 are based on the decay rate estimate ρ and the estimated variance of the decay regression residual $\hat{s}^2$ by (13). Less developed countries (LDC) as listed in the Supplementary Material (Section S.1). Nonmanufacturing merchandise spans SITC sector codes 0-4. Robust standard errors, clustered at the industry level and corrected for generated-regressor variation of export capability k, for ρ and s^2, applying the multivariate delta method to standard errors for η and σ. * marks significance at ten, ** at five, and *** at one-percent level.

tion in export capability around a common expected value for each country in each year due, say, to measurement error in trade data. If this measurement error were classical, all within-country variation in the exporter-industry fixed effects would be the result of iid disturbances that were uncorrelated across time. We would then observe no temporal connection between these distributions. When estimating the decay regression in (10), mean reversion would be complete, yielding a value of ρ close to -1. The coefficient estimates are inconsistent with such a pattern.

3.3 Comparative advantage as a stochastic process

On its own, reversion to the mean in log export capability is uninformative about the dynamics of its distribution.[24] While mean reversion is consistent with a stationary cross-sectional distribution, it is also consistent with a non-ergodic distribution or a degenerate comparative advantage that collapses at a long-term mean of one (log comparative advantage of zero). Degeneracy in comparative advantage is the interpretation that Levchenko and Zhang (2013) give to their finding of cross-country convergence in industry productivities. Yet, the combination of mean reversion in **Table 1** and temporal stability of the cumulative distribution plots in **Figure 2** is suggestive of a balance between random innovations to export capability and the dissipation of these capabilities. Such balance is characteristic of a stochastic process that generates a stationary cross-section distribution.[25]

To explore the dynamics of comparative advantage, we limit ourselves to the family of stochastic processes known as *diffusions*. Diffusions are Markov processes for which all realizations of the random variable are continuous functions of time and past realizations. We exploit the fact that the decay regression in (10) is consistent with the discretized version of a commonly studied diffusion, the Ornstein-Uhlenbeck (OU) process. Consider log comparative advantage $\ln \hat{A}_{is}(t)$—export capability normalized by industry-year and country-year means. Suppose that, when expressed in continuous time, comparative advantage $\hat{A}_{is}(t)$ follows an OU process given by

$$\mathrm{d} \ln \hat{A}_{is}(t) = -\frac{\eta\sigma^2}{2} \ln \hat{A}_{is}(t)\, \mathrm{d}t + \sigma\, \mathrm{d}W_{is}^{\hat{A}}(t), \tag{11}$$

where $W_{is}^{\hat{A}}(t)$ is a Wiener process that induces stochastic innovations in comparative advantage.[26] The parameter

[24] See, e.g., Quah's (1993, 1996) critique of using cross-country regressions to test for convergence in rates of economic growth.

[25] The underlying perpetual mean reversion of capability, and largely sector-invariant stochastic innovation, sit oddly with the notion that capability evolution is directed, such as from current industries to more sophisticated industries with related inputs (as posited, e.g., by Hidalgo et al. 2007).

[26] To relate equation (11) to trade theory, our specification for the evolution of export advantage is analogous to the equation of motion for a country's stock of ideas in the dynamic EK model of Buera and Oberfield (2016). In their model, each producer in source country s draws a productivity from a Pareto distribution, where this productivity combines multiplicatively with ideas learned from other firms, either within the same country or in different countries. Learning—or exposure to ideas—occurs at an exogenous rate $\alpha_s(t)$ and the transmissibility of ideas from one producer to another depends on the parameter β, which captures the transmissibility of ideas between producers. In equilibrium, the distribution of productivity across suppliers within a country is Fréchet, with location parameter equal to a country's current stock of ideas. The OU process in (11) emerges from the equation of motion for the stock of ideas in Buera and

η regulates the rate at which comparative advantage reverts to its global long-run mean and the parameter σ scales time and therefore the Brownian innovations $\mathrm{d}W^{\hat{A}}_{is}(t)$.[27] Because comparative advantage reflects a double normalization of export capability, it is natural to consider a global mean of zero for $\ln \hat{A}_{is}(t)$. The OU case is the unique non-degenerate Markov process that has a stationary normal distribution (Karlin and Taylor 1981). An OU process of log comparative advantage $\ln \hat{A}_{is}(t)$ therefore implies that $\hat{A}_{is}(t)$ has a stationary log normal distribution.

In (11), we refer to the parameter η as the *rate of dissipation* of comparative advantage because it contributes to the speed with which $\ln \hat{A}_{is}(t)$ would collapse to a degenerate level of zero if there were no stochastic innovations. The parametrization in (11) implies that η alone determines the shape of the stationary distribution, while σ is irrelevant for the cross section. Our parametrization treats η as a normalized rate of dissipation that measures the "number" of one-standard deviation shocks that dissipate per unit of time. We refer to σ as the *intensity of innovations*. It plays a dual role: on the one hand, σ governs volatility by scaling the Wiener innovations; on the other hand the parameter helps regulate the speed at which time elapses in the deterministic part of the diffusion.

To connect the continuous-time OU process in (11) to our decay regression in (10), we use the fact that the discrete-time process that results from sampling an OU process at a fixed time interval Δ is a Gaussian first-order autoregressive process with autoregressive parameter $\exp\{-\eta\sigma^2\Delta/2\}$ and innovation variance $(1 - \exp\{-\eta\sigma^2\Delta\})/\eta$ (Aït-Sahalia et al. 2010, Example 13). Applying this insight to the first-difference equation above, we obtain our decay regression:

$$k_{is}(t+\Delta) - k_{is}(t) = \rho\, k_{is}(t) + \delta_i(t) + \delta_s(t) + \varepsilon_{is}(t, t{+}\Delta), \tag{12}$$

which implies for the reduced-form decay parameter that

$$\rho \equiv -(1 - \exp\{-\eta\sigma^2\Delta/2\}) < 0,$$

for the unobserved country fixed effect that $\delta_s(t) \equiv \ln Z_s(t{+}\Delta) - (1{+}\rho) \ln Z_s(t)$, where $Z_s(t)$ is an arbitrary time-varying country-specific shock, and for the residual that $\varepsilon_{ist}(t, t{+}\Delta) \sim N\left(0,\ (1 - \exp\{-\eta\sigma^2\Delta\})/\eta\right)$.[28] An OU process with $\rho \in (-1, 0)$ generates a log normal stationary distribution in the cross section, with a shape

Oberfield (2016, equation (4)) as the limiting case with the transmissibility parameter $\beta \to 1$, provided that the learning rate $\alpha_s(t)$ is subject to random shocks and producers in a country only learn from suppliers within the same country. In Section 6, we discuss how equation (11) could be extended to allow for learning across national borders.

[27] Among possible parameterizations of the OU process, we choose (11) because it is related to our later extension to a generalized logistic diffusion and clarifies that the parameter σ is irrelevant for the shape of the cross-sectional distribution. We deliberately specify η and σ to be invariant over time, industry and country and in section 5 explore the goodness of fit under this restriction.

[28] For theoretical consistency, we state the country fixed effect $\delta_s(t)$ as a function of the shock $Z_s(t)$, which we will formally define as a country-wide stochastic trend in equation (14) below and then identify in subsequent GMM estimation.

parameter of $1/\eta$ and a mean of zero.

The reduced-form decay coefficient ρ in (12) is a function both of the dissipation rate η and the intensity of innovations σ and may differ across samples because either or both of those parameters vary. This distinction is important because ρ may vary even if the shape of the distribution of comparative advantage does not change.[29] From OLS estimation of (12), we can obtain estimates of η and σ^2 using the solutions,

$$\begin{aligned} \eta &= \frac{1-(1+\hat{\rho})^2}{\hat{s}^2} \\ \sigma^2 &= \frac{\hat{s}^2}{1-(1+\hat{\rho})^2} \frac{\ln(1+\hat{\rho})^{-2}}{\Delta}, \end{aligned} \tag{13}$$

where $\hat{\rho}$ is the estimated decay rate and $\hat{s}^2$ is the estimated variance of the decay regression residual.

Table 1 shows estimates of η and σ^2 implied by the decay regression results, with standard errors obtained using the multivariate delta method.[30] The estimate of η based on OLS export capability, at 0.28 in column 1 of **Table 1**, is larger than those based on PPML export capability, at 0.20 in column 4, or the log RCA index, at 0.22 in column 7, implying that the distribution of OLS export capability will be more concave to the origin. But estimates generally indicate strong concavity, consistent with the visual evidence in **Figure 2**. To gain intuition about η, suppose the intensity of innovations of the Wiener process is unity ($\sigma = 1$). Then a value of η equal to 0.28 means that it will take 5.0 years for half of the initial shock to log comparative advantage to dissipate (and 16.4 years for 90% of the initial shock to dissipate). Alternatively, if η equals 0.20 it will take 6.9 years for half of the initial shock to decay (and 23.0 years for 90% of the initial shock to dissipate).[31]

To see how the dissipation rate and the innovation intensity affect the reduced-form decay parameter ρ, we compare η and σ^2 across subsamples. First, compare the estimate for ρ in the subsample of developing economies in column 2 of **Table 1** to that for the full sample of countries in column 1. The larger estimate of ρ in the former sample (-0.45 in column 2 versus -0.35 in column 1) implies that *reduced-form* mean reversion is relatively rapid in developing countries. However, this result is silent about how the shape of the distribution of comparative advantage varies across nations. The absence of a statistically significant difference in the estimated dissipation rate η between the developing-country sample ($\eta = 0.29$) and the full-country sample ($\eta = 0.28$) indicates that comparative advantage is similarly heavy-tailed in the two groups. The larger reduced-form decay rate ρ for

[29]The estimated value of ρ is sensitive to the time interval $\triangle$ that we define in (12), whereas the estimated value of η is not. At shorter time differences—for which there may be relatively more noise in export capability—the estimated magnitude of σ is larger and therefore the reduced-form decay parameter ρ is as well. However, the estimated intrinsic speed of mean reversion η is unaffected. In unreported results, we verify these insights by estimating the decay regression in (10) for time differences of 1, 5, 10, and 15 years.

[30]Details on the construction of standard errors for η and σ^2 are available in the Supplementary Material (Section S.3).

[31]In the absence of shocks and for $\sigma = 1$, log comparative advantage follows the deterministic differential equation $\mathrm{d} \ln \hat{A}_{is}(t) = -(\eta/2) \ln \hat{A}_{is}(t)\, \mathrm{d}t$ by (16) and Itō's lemma, with the solution $\ln \hat{A}_{is}(t) = \ln \hat{A}_{is}(0) \exp\{-(\eta/2)t\}$. Therefore, the number of years for a dissipation of $\ln \hat{A}_{is}(0)$ to a remaining level $\ln \hat{A}_{is}(T)$ is $T = 2 \log[\ln \hat{A}_{is}(0)/ \ln \hat{A}_{is}(T)]/\eta$.

developing countries results from their having a larger intensity of innovations ($\sigma = 0.64$ in column 2 versus $\sigma = 0.56$ in column 1, where this difference is statistically significant). In other words, a one-standard-deviation shock to comparative advantage in a developing country dissipates at roughly the same rate as in an industrialized country. But because the magnitude of this shock is larger for the developing country, its observed rate of decay will be faster (otherwise the country's export capabilities would not have a stationary cross-sectional distribution).

Second, compare nonmanufacturing industries in column 3 to the full sample of industries in column 1. Whereas the average nonmanufacturing industry differs from the average overall industry in the reduced-form decay rate ρ (-0.45 in column 3 versus -0.35 in column 1), it shows no such difference in the estimated dissipation rate η (0.28 in column 1 versus 0.29 in column 3). This implies that comparative advantage has comparably heavy tails among manufacturing and nonmanufacturing industries. However, the intensity of innovations σ is larger for nonmanufacturing industries (0.65 in column 3 versus 0.56 in column 1), due perhaps to higher output volatility associated with resource discoveries. These nuances regarding the implied shape of and the convergence speed toward the cross-sectional distribution of comparative advantage are undetectable when one considers the reduced-form decay rate ρ alone.[32]

The diffusion model in (11) and its discrete-time analogue in (12) reveal a deep connection between heavy tails in export advantage and churning in industry export ranks. Random innovations in absolute advantage cause industries to alternate positions in the cross-sectional distribution of comparative advantage for a country at a rate of innovation precisely fast enough so that the deterministic dissipation of absolute advantage creates a stable, heavy-tailed distribution of export prowess. Having established the plausibility of comparative advantage as following a stochastic process, we turn next to a more rigorous analysis of the properties of this process.

4 The Diffusion of Comparative Advantage

The OU process is but one of many that would yield a stationary distribution for comparative advantage that has heavy tails. In this section, we stay within the family of diffusions but define a generalized logistic diffusion for comparative advantage, which includes the OU process as a limiting case. The GLD that we specify below allows us to test the OU process against well-defined alternatives, to evaluate the fit of the model to the data, and to characterize the dynamic implications of the model, all of which we undertake in Section 5.

[32] Appendix **Table A1** shows results for two- and three-digit industries for the subperiod 1984-2007. Whereas reduced-form decay rates ρ increase in magnitude as one goes from the two- to the three-digit level, dissipation rates η remain stable. The difference in reduced-form decay rates ρ is driven by a higher intensity of innovations σ among the more narrowly defined three-digit industries. Intuitively, the magnitude of shocks to comparative advantage is larger in the more disaggregated product groupings.

4.1 Generalized logistic diffusion

In **Figures A1** through **A3** in the Appendix, which show the cumulative distributions from **Figure 2** over time, the cross-sectional distributions of absolute advantage shift rightward for each country over time, consistent with the series being non-stationary. Yet, the cross-sectional distributions preserve their shape across periods, which suggests that once we adjust absolute advantage for country-wide productivity growth, we obtain a stationary series. We define this series to be *generalized comparative advantage*, written in continuous time as,

$$\hat{A}_{is}(t) \equiv \frac{A_{is}(t)}{Z_s(t)}, \tag{14}$$

where $A_{is}(t)$ is observed absolute advantage and $Z_s(t)$ is an unobserved country-wide stochastic trend (an arbitrary country-specific shock to absolute advantage).[33] The relationship between comparative and absolute advantage in equation (14) highlights an important difference between economy-wide growth, reflected in a country's mean absolute advantage, and trade specialization, reflected in comparative advantage. The time-varying country-specific shock $Z_s(t)$ may exhibit systematic covariation with specific industries' absolute advantages $A_is(t)$ but will be inconsequential for the stochastic evolution of comparative advantage. We specify that comparative advantage in (14) follows a generalized logistic diffusion (GLD).

We provide a formal derivation of the GLD and its stationary distribution in Appendix A. The stationary distribution of the GLD is the generalized gamma, which unifies the gamma and extreme-value distributions, as well as many others (Crooks 2010), and has the log normal, the Pareto, and other commonly used distributions as special or limiting cases. To motivate our choice of the GLD, and hence of the generalized gamma as the cross-sectional distribution for comparative advantage, consider the graphs in **Figure 2** (as well as **Figures A1** through **A3** in the Appendix). These figures are broadly consistent with comparative advantage being log normal. But they also indicate that for many countries the number of industries drops off more quickly or more slowly in the upper tail than the log normal distribution can capture. The generalized gamma distribution accommodates such kurtosis.[34]

Formally, in a cross section of the data after arbitrarily much time has passed, the generalized gamma probability density function for a realization $\hat{a}_{is}$ of the random variable comparative advantage $\hat{A}_{is}$ is given by:

$$f_{\hat{A}}(\hat{a}_{is}|\hat{\theta}, \kappa, \phi) = \frac{1}{\Gamma(\kappa)} \left|\frac{\phi}{\hat{\theta}}\right| \left(\frac{\hat{a}_{is}}{\hat{\theta}}\right)^{\phi\kappa-1} \exp\left\{-\left(\frac{\hat{a}_{is}}{\hat{\theta}}\right)^{\phi}\right\} \qquad \text{for} \quad \hat{a}_{is} > 0, \tag{15}$$

[33] This measure satisfies the properties of comparative advantage in (5), which compares country and industry pairs.

[34] Our implementation of the generalized gamma uses three parameters, as in Stacy (1962). In their analysis of the firm size distribution by age, Cabral and Mata (2003) also use a version of the generalized gamma distribution with a support bounded below by zero and document a good fit.

where $\Gamma(\cdot)$ denotes the gamma function and $(\hat{\theta}, \kappa, \phi)$ are real parameters with $\hat{\theta}, \kappa > 0$.[35] The generalized gamma nests the ordinary gamma distribution for $\phi = 1$ and the log normal or Pareto distributions when ϕ tends to zero.[36] For real parameters (η, σ, ϕ), the generalized logistic diffusion,

$$\frac{\mathrm{d}\hat{A}_{is}(t)}{\hat{A}_{is}(t)} = \frac{\sigma^2}{2}\left[1 - \eta\,\frac{\hat{A}_{is}(t)^{\phi} - 1}{\phi}\right]\mathrm{d}t + \sigma\,\mathrm{d}W^{\hat{A}}_{is}(t), \tag{16}$$

has a stationary distribution that is generalized gamma with a probability density $f_{\hat{A}}(\hat{a}_{is}|\hat{\theta}, \kappa, \phi)$ given by (15) and the real parameters

$$\hat{\theta} = \left(\phi^2/\eta\right)^{1/\phi} > 0 \quad \text{and} \quad \kappa = 1/\hat{\theta}^{\phi} > 0.$$

A non-degenerate stationary distribution exists only if $\eta > 0$.[37] The variable $W^{\hat{A}}_{is}(t)$ is the Wiener process. The GLD nests the OU process as $\phi \to 0$ (with η finite).

The term $(\sigma^2/2)[1 - \eta\{\hat{A}_{is}(t)^{\phi} - 1\}/\phi]$ in (16) is a deterministic drift that regulates the relative change in comparative advantage $\mathrm{d}\hat{A}_{is}(t)/\hat{A}_{is}(t)$. It involves both constant parameters (η, σ, ϕ) and a level-dependent component $\hat{A}_{is}(t)^{\phi}$, where ϕ is the elasticity of the mean reversion with respect to the current level of absolute advantage. We call ϕ the *elasticity of decay*. For the OU process ($\phi \to 0$), the relative change in absolute advantage is neutral with respect to the current level. If $\phi > 0$, then the level-dependent drift component $\hat{A}_{is}(t)^{\phi}$ leads to a faster than neutral mean reversion from above than from below the mean, indicating that the loss of absolute advantage in a currently strong industry tends to occur more rapidly than the buildup of absolute disadvantage in a currently weak industry. Conversely, if $\phi < 0$ then mean reversion tends to occur more slowly from above than below the long-run mean.

The parameters η and σ in (16) inherit their interpretations from the OU process in (11) as the rate of dissipation and the intensity of innovations. Under the GLD, the dissipation rate η and decay elasticity ϕ jointly determine the heavy tail of the cross-sectional distribution of comparative advantage, with the intensity of innovations σ regulating the speed of convergence to this distribution but having no effect on its shape. For subsequent

[35]We do not restrict ϕ to be strictly positive (as do e.g. Kotz et al. 1994, ch. 17). We allow ϕ to take any real value (see Crooks 2010), including a strictly negative ϕ for a generalized inverse gamma distribution. Crooks (2010) shows that this generalized gamma distribution (Amoroso distribution) nests the Fréchet, Weibull, gamma, inverse gamma and numerous other distributions as special cases and yields the normal, log normal and Pareto distributions as limiting cases.

[36]As ϕ goes to zero, it depends on the limiting behavior of κ whether a log normal distribution or a Pareto distribution results (Crooks 2010, Table 1). The parameter restriction $\phi = 1$ clarifies that the generalized gamma distribution results when one takes an ordinary gamma distributed variable and raises it to a finite power $1/\phi$.

[37]In the estimation, we will impose the constraint that $\eta > 0$. If η were negative, comparative advantage would collapse over time for $\phi < 0$ or explode for $\phi \geq 0$. We do not constrain η to be finite.

derivations, it is convenient to restate the GLD (16) more compactly in terms of log changes as,

$$\mathrm{d}\ln \hat{A}_{is}(t) = -\frac{\eta\sigma^2}{2}\frac{\hat{A}_{is}(t)^{\phi}-1}{\phi}\,\mathrm{d}t + \sigma\,\mathrm{d}W_{is}^{\hat{A}}(t), \tag{17}$$

which follows from (16) by Itō's lemma.[38]

4.2 The cross-sectional distributions of comparative and absolute advantage

Absolute advantage, defined as in (3), is measurable by exporter-sector-year fixed effects estimated from the gravity model in (6). In contrast, generalized comparative advantage, as defined in (14), has an unobserved country-specific stochastic trend $Z_s(t)$, and lacks a direct empirical counterpart. We therefore need to identify $Z_s(t)$ in estimation. Intuitively, identification of the country-level shocks $Z_s(t)$ is possible because we can observe the evolving position of the cumulative absolute advantage distribution over time and, as we now show, the evolving position is the only difference between the cumulative distributions of absolute and comparative advantage.

The stationary distribution of absolute advantage is closely related to that of comparative advantage under the maintained assumption that comparative advantage $\hat{A}_{is}(t)$ follows a generalized logistic diffusion given by (16). As stated before, the GLD of comparative advantage implies that the stationary distribution of comparative advantage $\hat{A}_{is}(t)$ is generalized gamma with the CDF

$$F_{\hat{A}}(\hat{a}_{is}|\hat{\theta},\phi,\kappa) = G\left[\left(\frac{\hat{a}_{is}}{\hat{\theta}}\right)^{\phi};\kappa\right],$$

where $G[x;\kappa] \equiv \gamma_x(\kappa;x)/\Gamma(\kappa)$ is the ratio of the lower incomplete gamma function and the gamma function. As we show in Appendix A.3, then the cross-sectional distribution of absolute advantage $A_{is}(t)$ is also generalized gamma but with the CDF

$$F_A(a_{is}|\theta_s(t),\phi,\kappa) = G\left[\left(\frac{a_{is}}{\theta_s(t)}\right)^{\phi};\kappa\right]$$

for the strictly positive parameters

$$\hat{\theta} = \left(\phi^2/\eta\right)^{1/\phi}, \quad \theta_s(t) = \hat{\theta}Z_s(t) \quad \text{and} \quad \kappa = 1/\hat{\theta}^{\phi}.$$

[38]Returning to the connection between our estimation and the dynamic EK model in Buera and Oberfield (2016)—also see footnotes 9 and 26—the specification in (16) is equivalent to their equation of motion for the stock of ideas (Buera and Oberfield 2016, equation (4)) under the assumptions that producers only learn from suppliers within their national borders and the learning rate $\alpha_s(t)$ is constantly growing across industries, countries, and over time but subject to idiosyncratic shocks. The parameter ϕ in (16) is equivalent to the value $\beta - 1$ in their model, where β captures the transmissibility of ideas between producers. Our finding, discussed in Section 5, that ϕ is small and negative implies that the value of β in the Buera and Oberfield (2016) model is large (but just below 1, as they require).

These cumulative distribution functions follow from Kotz et al. (1994, Ch. 17, Section 8.7).

The two cross sectional distributions of comparative and absolute advantage differ only in the scale parameter. For comparative advantage, the scale parameter $\hat{\theta}$ is time invariant. For absolute advantage, the scale parameter $\theta_s(t) = \hat{\theta} Z_s(t)$ is time varying (and potentially stochastic) but country specific (industry invariant). Empirically, $\theta_s(t)$ typically increases over time so that, visually, the plotted cumulative distributions of absolute advantage shift rightward over time (as can bee seen from a comparison of the cumulative distribution plots for 1967, 1987 and 2007 in Appendix **Figures A1, A2** and **A3**).

This connection between the cumulative distributions of absolute and comparative advantage allows us to estimate a GLD for generalized comparative advantage based on data for absolute advantage alone. The mean of the log of the distribution of absolute advantage can be calculated explicitly as a function of the model parameters, enabling us to identify the trend from the relation that $\mathbb{E}_{st}[\ln \hat{A}_{is}(t)] = \mathbb{E}_{st}[\ln A_{is}(t)] - \ln Z_s(t)$, which follows by definition (14).[39] As we show in Appendix B, if comparative advantage $\hat{A}_{is}(t)$ follows the generalized logistic diffusion (16), then the country specific stochastic trend $Z_s(t)$ can be identified from the first moment of the logarithm of absolute advantage as:

$$Z_s(t) = \exp\left\{\mathbb{E}_{st}[\ln A_{is}(t)] - \frac{\ln(\phi^2/\eta) + \Gamma'(\eta/\phi^2)/\Gamma(\eta/\phi^2)}{\phi}\right\} \tag{18}$$

where $\Gamma'(\kappa)/\Gamma(\kappa)$ is the digamma function. We obtain detrended comparative advantage measures based on the sample analog of equation (18):

$$\hat{A}_{is}(t) = \exp\left\{\ln A_{is}(t) - \frac{1}{I}\sum_{j=1}^{I} \ln A_{js}(t) + \frac{\ln(\phi^2/\eta) + \Gamma'(\eta/\phi^2)/\Gamma(\eta/\phi^2)}{\phi}\right\} \tag{19}$$

This result allows us to estimate the GLD of comparative advantage $\hat{A}_{is}(t)$ using absolute advantage data $A_{is}(t)$.

4.3 A GMM estimator

The generalized logistic diffusion model (16) has no known closed-form transition density when $\phi \neq 0$. We therefore cannot evaluate the likelihood of the observed data and cannot perform maximum likelihood estimation. However, an attractive feature of the GLD is that it can be transformed into a stochastic process that belongs to the Pearson-Wong family, for which closed-form solutions of the conditional moments do exist.[40] As documented in detail in Appendix C, we construct a consistent GMM estimator based on the conditional moments of a

[39]The expectations operator $\mathbb{E}_{st}[\cdot]$ denotes the conditional expectation for source country s at time t.

[40]Pearson (1895) first studied the family of distributions now called Pearson distributions. Wong (1964) showed that the Pearson distributions are stationary distributions of a specific class of stochastic processes, for which conditional moments exist in closed form.

transformation of comparative advantage, using results from Forman and Sørensen (2008).

Formally, if comparative advantage $\hat{A}_{is}(t)$ follows the generalized logistic diffusion (16) with real parameters η, σ, ϕ ($\eta > 0$), then the transformed variable

$$\hat{B}_{is}(t) = [\hat{A}_{is}(t)^{-\phi} - 1]/\phi \tag{20}$$

follows the diffusion

$$\mathrm{d}\hat{B}_{is}(t) = -\frac{\sigma^2}{2}\left[\left(\eta - \phi^2\right)\hat{B}_{is}(t) - \phi\right]\mathrm{d}t + \sigma\sqrt{\phi^2\hat{B}_{is}(t)^2 + 2\phi\hat{B}_{is}(t) + 1}\;\mathrm{d}W_{is}^{\hat{B}}(t)$$

and belongs to the Pearson-Wong family (see Appendix C.1 for the derivation). As elaborated in Appendix C.2, it is then possible to recursively derive the n-th conditional moment of the transformed process $\hat{B}_{is}(t)$ and to calculate a closed form for the conditional moments of the transformed process at time t_τ conditional on the information set at time $t_{\tau-1}$. If we use these conditional moments to forecast the m-th power of $\hat{B}_{is}(t_\tau)$ with time $t_{\tau-1}$ information, the resulting forecast errors are uncorrelated with any function of past $\hat{B}_{is}(t_{\tau-1})$. We can therefore construct a GMM criterion for estimation. Denote the forecast error with

$$U_{is}(m, t_{\tau-1}, t_\tau) = \hat{B}_{is}(t_\tau)^m - \mathbb{E}\left[\hat{B}_{is}(t_\tau)^m \,\middle|\, \hat{B}_{is}(t_{\tau-1})\right].$$

This random variable represents an unpredictable innovation in the m-th power of $\hat{B}_{is}(t_\tau)$. As a result, the forecast error $U_{is}(m, t_{\tau-1}, t_\tau)$ is uncorrelated with any measurable transformation of $\hat{B}_{is}(t_{\tau-1})$.

A GMM criterion function based on these forecast errors is

$$g_{is\tau}(\eta, \sigma, \phi) \equiv [h_1(\hat{B}_{is}(t_{\tau-1}))U_{is}(1, t_{\tau-1}, t_\tau), \ldots, h_M(\hat{B}_{is}(t_{\tau-1}))U_{is}(M, t_{\tau-1}, t_\tau)]',$$

where each h_m is a row vector of measurable functions specifying instruments for the m-th moment condition. This criterion function has mean zero due to the orthogonality between the forecast errors and the time $t_{\tau-1}$ instruments. Implementing GMM requires a choice of instruments. Computational considerations lead us to choose polynomial vector instruments of the form $h_m(\hat{B}_{is}(t)) = (1, \hat{B}_{is}(t), \ldots, \hat{B}_{is}(t)^{K-1})'$ to construct K instruments for each of the M moments that we include in our GMM criterion.[41] In the estimation, we use $K = 2$ instruments and $M = 2$ conditional moments, providing us with $K \cdot M = 4$ equations and overidentifying the

[41] We work with a suboptimal estimator because the optimal-instrument GMM estimator considered by Forman and Sørensen (2008) requires the inversion of a matrix for each observation. Given our large sample, this task is numerically expensive. Also, we found local minima in our GMM criterion. At the cost of additional computation, we use a global optimization algorithm to find our estimates of ϕ, η, and σ^2. Specifically, we use Matlab's Genetic Algorithm included in the Global Optimization Toolbox. These computational concerns lead us to sacrifice efficiency and use suboptimal instruments.

three parameters (η, σ, ϕ). Appendix C.3 gives further details on our GMM routine.

Standard errors of our estimates need to account for the preceding estimation of our absolute advantage $\ln A_{is}(t)$ measures. Newey and McFadden (1994) present a two-step estimation method for GMM, which accounts for the presence of generated (second-stage) variables that are predicted (from a first stage). However, our absolute advantage $\ln A_{is}(t)$ measures are not predicted variables but parameter estimates from a gravity equation: $\ln A_{is}(t)$ is a normalized version of the estimated exporter-sector-year fixed effect in equations (6) and (8). Whereas the Newey-McFadden results require a constant number of first-stage parameters, the number of parameters we estimate in our first stage increases with our first-stage sample size. Moreover, the moments in GMM time series estimation here (just as the variables in OLS decay estimation in Section 3.2 above) involve pairs of parameter estimates from different points in time—$\ln A_{is}(t)$ and $\ln A_{is}(t + \Delta)$—and thus require additional treatments of induced covariation in the estimation. In Appendix D, we extend Newey and McFadden (1994) to our specific finite-sample context, which leads to an alternative two-step estimation method that we employ for the computation of standard errors. We use the multivariate delta method to calculate standard errors for transformed functions of the estimated parameters.

5 Estimates

Following the GMM procedure described in Section 4.3, we estimate the dissipation rate η, innovation intensity σ, and decay elasticity ϕ in the diffusion of comparative advantage, subject to an estimated country-specific stochastic trend $Z_s(t)$. The trend allows absolute advantage to be non-stationary but, because it is common to all industries in a country, the trend has no bearing on comparative advantage. Estimating the GLD permits us to test the strong distributional assumptions implicit in the OLS estimation of the discretized OU process and to evaluate the fit of the model, with or without the OU restrictions applied.

5.1 GMM results for the Generalized Logistic Diffusion

Table 2 presents our baseline GMM estimation results using moments on five-year intervals. We move to a five-year horizon, from the ten-year horizon in the OLS decay regressions in **Table 1**, to allow for a more complete description of the time-series dynamics of comparative advantage. For robustness, we also report GMM results using moments on ten-year intervals (see the Supplementary Material, Table S3). Similar to the OLS decay regressions, we use measures of export advantage based on OLS gravity estimates of export capability, PPML gravity estimates of export capability, and the Balassa RCA index.

The key distinction between the OU process in (11) and the GLD in (17) is the presence of the decay elasticity

Table 2: GMM Estimates of Comparative Advantage Diffusion, 5-year Transitions

	OLS gravity k			PPML gravity k			$\ln RCA$		
	All	LDC	Nonmanf.	All	LDC	Nonmanf.	All	LDC	Nonmanf.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Estimated Generalized Logistic Diffusion Parameters									
Dissipation rate η	0.256	0.270	0.251	0.180	0.166	0.147	0.212	0.194	0.174
	(0.004)***	(0.006)***	(0.005)***	(0.006)***	(0.004)***	(0.005)***	(0.006)***	(0.012)***	(0.008)***
Intensity of innovations σ	0.739	0.836	0.864	0.767	0.863	0.852	0.713	0.789	0.722
	(0.01)***	(0.017)***	(0.017)***	(0.037)***	(0.03)***	(0.045)***	(0.051)***	(0.082)***	(0.042)***
Elasticity of decay ϕ	-0.041	-0.071	-0.033	-0.009	-0.002	-0.006	0.006	-0.011	-0.045
	(0.017)**	(0.027)***	(0.018)*	(0.035)	(0.028)	(0.038)	(0.053)	(0.083)	(0.039)
Implied Parameters									
Log gen. gamma scale $\ln\hat{\theta}$	121.94	56.50	164.79	900.95	6,122.90	1,425.40	-1,410.50	708.56	99.83
	(71.526)*	(32.175)*	(120.946)	(4581.812)	(113520.900)	(11449.450)	(14980.320)	(7069.866)	(126.167)
Log gen. gamma shape $\ln\kappa$	5.017	3.991	5.439	7.788	10.873	8.360	8.641	7.467	4.464
	(0.842)***	(0.76)***	(1.077)***	(8.062)	(31.199)	(12.926)	(17.289)	(15.685)	(1.714)***
Mean/median ratio	8.203	8.203	8.293	16.897	20.469	31.716	10.256	13.872	25.286
Observations	392,850	250,300	190,630	389,290	248,360	187,390	392,860	250,300	190,630
Industry-source obs. $I \times S$	11,542	7,853	5,845	11,531	7,843	5,835	11,542	7,853	5,845
Root mean sq. forecast error	1.851	2.028	1.958	1.898	2.026	2.013	1.760	1.930	1.965
Min. GMM obj. ($\times$ 1,000)	3.27e-13	7.75e-13	7.37e-13	2.56e-12	7.82e-12	5.53e-12	6.79e-12	2.16e-11	1.65e-11

Source: WTF (Feenstra et al. 2005, updated through 2008) for 133 time-consistent industries in 90 countries from 1962-2007 and CEPII.org; OLS and PPML gravity measures of export capability (log absolute advantage) $k = \ln A$ from (8).
Note: GMM estimation at the five-year horizon for the generalized logistic diffusion of comparative advantage $\hat{A}_{is}(t)$,

$$d\ln\hat{A}_{is}(t) = -\frac{\eta\sigma^2}{2}\frac{\hat{A}_{is}(t)^{\phi}-1}{\phi}\,dt + \sigma\, dW^{\hat{A}}_{is}(t)$$

using absolute advantage $A_{is}(t) = \hat{A}_{is}(t)Z_s(t)$ based on OLS and PPML gravity measures of export capability k from (6) and (8), and the Balassa index of revealed comparative advantage $RCA_{ist} = (X_{ist}/\sum_{\varsigma} X_{i\varsigma t})/(\sum_j X_{jst}/\sum_j \sum_{\varsigma} X_{j\varsigma t})$. Parameters η, σ, ϕ are estimated under the constraints $\ln\eta, \ln\sigma^2 > -\infty$ for the mirror Pearson (1895) diffusion of (20), while concentrating out country-specific trends $Z_s(t)$. The implied parameters are inferred as $\hat{\theta} = (\phi^2/\eta)^{1/\phi}$, $\kappa = 1/\hat{\theta}^{\phi}$and the mean/median ratio is given by (A.10). Less developed countries (LDC) as listed in the Supplementary Material (Section S.1). The manufacturing sector spans SITC one-digit codes 5-8, the nonmanufacturing merchandise sector codes 0-4. Robust errors in parentheses (corrected for generated-regressor variation of export capability k): * marks significance at ten, ** at five, and *** at one-percent level. Standard errors of transformed and implied parameters are computed using the multivariate delta method.

ϕ, which allows for asymmetry in mean reversion from above versus below the mean. The sign of ϕ captures the nature of this asymmetry. Using OLS gravity estimates of comparative advantage (columns 1 to 3 in **Table 2**), the GMM estimate of ϕ is negative and statistically significant. Negativity in ϕ implies that comparative advantage reverts to the long-run mean more slowly from above than from below. Industries that randomly churn into the upper tail of the cross section will tend to retain their comparative advantage for longer than those below the mean, affording high-advantage industries with opportunities to reach higher levels of comparative advantage as additional innovations arrive. Thus, for OLS gravity estimates of comparative advantage we reject log normality in favor of the generalized gamma distribution.

The rejection of log normality, however, is not robust across measures of comparative advantage. In **Table 2**, using PPML gravity estimates of comparative advantage (columns 4 to 6) or the Balassa RCA index (columns 7 to 9) produces GMM estimates of ϕ that are statistically insignificant and small in magnitude.[42] These results are an initial indication that imposing log normality on comparative advantage may not be a grave abuse of reality. A second indication that imposing log normality may not be unwarranted is that GMM estimates of the dissipation rate η for the GLD in **Table 2** are similar to those derived from the OLS decay regression in **Table 1**. In both sets of results, the dissipation rate η takes a value of about one-quarter for OLS gravity comparative advantage, about one-sixth for PPML gravity comparative advantage, and about one-fifth for the Balassa RCA index.

To make precise comparisons of parameter estimates under alternative distribution assumptions for comparative advantage, in **Table 3** we report GMM results (for OLS gravity estimates of comparative advantage) with and without imposing the restriction that $\phi = 0$. Without this restriction (columns 1, 3, 5 and 7), we allow comparative advantage to have a generalized gamma distribution; with this restriction (columns 2, 4, 6, and 8), we impose log normality. Estimates for the dissipation rate η and the innovation intensity σ are nearly identical in each pair of columns. This parameter stability implies that the special case of the OU process captures the broad persistence and overall variability of comparative advantage. Because the decay elasticity ϕ also determines the shape of the stationary distribution of the GLD, two processes that have identical values of η but distinct values of ϕ will differ in the shape of their generalized gamma distributions. We see in **Table 3** that the implied mean/median ratios are modestly higher for columns where ϕ is unrestricted (and found to be small and negative) versus columns in which ϕ is set to zero. The estimated mean-median ratio increases from $6.2 - 7.0$ under the constrained estimation of the OU process to $8.2 - 8.3$ under the unconstrained case. The extension to a GLD thus appears to help explain the export concentration in the upper tail documented in **Section 3.1**.

Table 3 also allows us to see the impact on the GMM parameter estimates of altering the time interval on which moment conditions are based. Columns 7 and 8 show results for moments on ten-year intervals, which

[42] As shown in Appendix **Table A2**, we obtain similar results for the 1984 to 2007 period when we use two- or three-digit SITC revision 2 industries, thus establishing the robustness of the GMM results under alternative industry aggregation.

Table 3: GMM ESTIMATES OF COMPARATIVE ADVANTAGE DIFFUSION, UNRESTRICTED AND RESTRICTED

	OLS gravity k, 5-year transitions						OLS k, 10-yr. trans.	
	Full sample		LDC exp.		Non-manuf.		Full sample	
		$\phi = 0$		$\phi = 0$		$\phi = 0$		$\phi = 0$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Estimated Generalized Logistic Diffusion Parameters								
Dissipation rate η	0.256	0.263	0.270	0.274	0.251	0.256	0.264	0.265
	(0.004)***	(0.003)***	(0.006)***	(0.003)***	(0.005)***	(0.004)***	(0.004)***	(0.003)***
Intensity of innovations σ	0.739	0.736	0.836	0.831	0.864	0.860	0.569	0.568
	(0.01)***	(0.008)***	(0.017)***	(0.012)***	(0.017)***	(0.014)***	(0.007)***	(0.006)***
Elasticity of decay ϕ	-0.041		-0.071		-0.033		-0.029	
	(0.017)**		(0.027)***		(0.018)*		(0.014)**	
Implied Parameters								
Log gen. gamma scale $\ln \hat{\theta}$	121.940		56.502		164.790		202.430	
	(71.526)*		(32.175)*		(120.946)		(131.469)	
Log gen. gamma shape $\ln \kappa$	5.017		3.991		5.439		5.781	
	(0.842)***		(0.76)***		(1.077)***		(0.968)***	
Mean/median ratio	8.203	6.691	8.203	6.222	8.293	7.036	7.281	6.588
Observations	392,850	392,850	250,300	250,300	190,630	190,630	335,820	335,820
Industry-source obs. $I \times S$	11,542	11,542	7,853	7,853	5,845	5,845	11,213	11,213
Root mean sq. forecast error	1.851	1.726	2.028	1.821	1.958	1.859	1.876	1.799
Min. GMM obj. ($\times$ 1,000)	3.27e-13	2.87e-12	7.75e-13	2.08e-11	7.37e-13	8.99e-12	3.03e-12	5.92e-12

Source: WTF (Feenstra et al. 2005, updated through 2008) for 133 time-consistent industries in 90 countries from 1962-2007 and CEPII.org; OLS gravity measures of export capability (log absolute advantage) $k = \ln A$ from (6).
Note: GMM estimation at the five-year (ten-year) horizon for the generalized logistic diffusion of comparative advantage $\hat{A}_{is}(t)$,

$$\mathrm{d} \ln \hat{A}_{is}(t) = -\frac{\eta\sigma^2}{2} \frac{\hat{A}_{is}(t)^{\phi} - 1}{\phi} \,\mathrm{d}t + \sigma \,\mathrm{d}W^{\hat{A}}_{is}(t)$$

using absolute advantage $A_{is}(t) = \hat{A}_{is}(t)Z_s(t)$, unrestricted and restricted to $\phi = 0$. Parameters η, σ, ϕ are estimated under the constraints $\ln \eta, \ln \sigma^2 > -\infty$ for the mirror Pearson (1895) diffusion of (20), while concentrating out country-specific trends $Z_s(t)$. The implied parameters are inferred as $\hat{\theta} = (\phi^2/\eta)^{1/\phi}$, $\kappa = 1/\hat{\theta}^{\phi}$and the mean/median ratio is given by (A.10). Less developed countries (LDC) as listed in the Supplementary Material (Section S.1). The manufacturing sector spans SITC one-digit codes 5-8, the nonmanufacturing merchandise sector codes 0-4. Robust errors in parentheses (corrected for generated-regressor variation of export capability k): * marks significance at ten, ** at five, and *** at one-percent level. Standard errors of transformed and implied parameters are computed using the multivariate delta method.

compare to the preceding columns whose results are for moments on five-year intervals.[43] Whereas estimates for the dissipation rate η are nearly identical for the two time horizons, estimates for the innovation intensity σ become smaller when we move from five-year to ten-year intervals. Similar to attenuation bias driving estimates of persistence to zero in auto-regression models, measurement error appears to deliver larger values of σ at shorter horizons.[44]

5.2 Model fit I: Matching dynamic transition probabilities

We next evaluate the dynamic performance of the model by assessing how well the GLD replicates the churning of export industries observed in the data. Using estimates based on the five-year horizon from column 1 in **Table** 2, we simulate trajectories of the GLD. In the simulations, we predict the model's transition probabilities over the one-year horizon across percentiles of the cross-section distribution. We deliberately use a shorter time horizon for the simulation than the five-year horizon used for estimation to assess moments that we did not target in the GMM routine. We then compare the model-based predictions to the empirical transition probabilities at the one-year horizon.

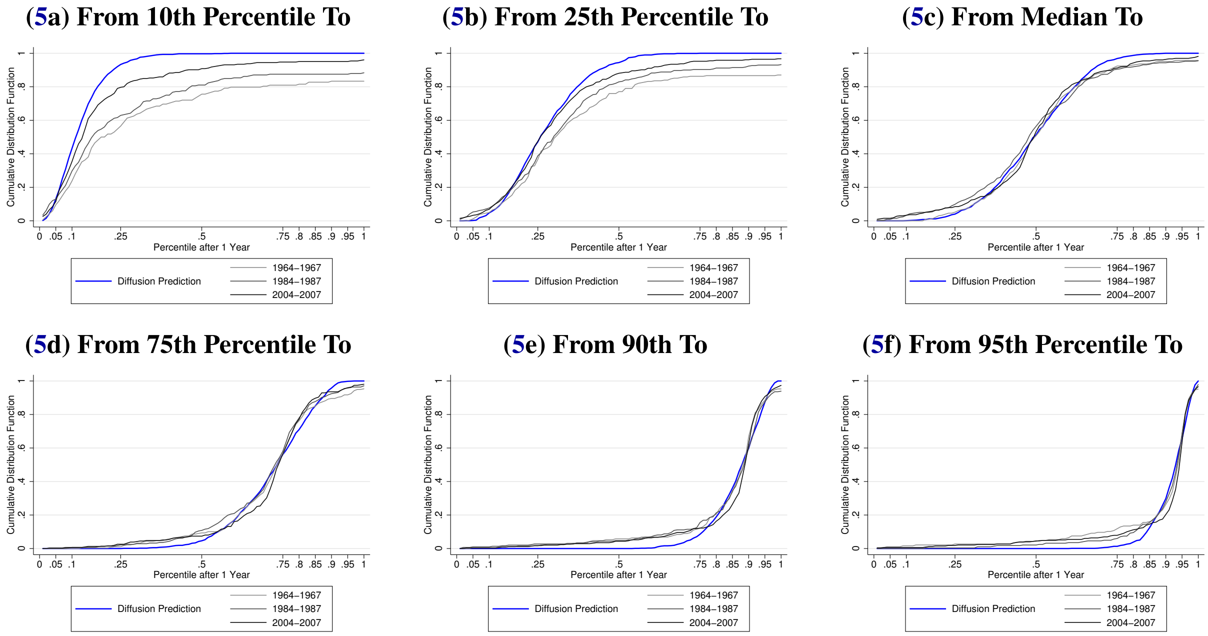
Figure 5 shows empirical and model-predicted conditional cumulative distribution functions for annual transitions of comparative advantage. We pick select percentiles in the start year: the 10th and 25th percentile, the median, the 75th, 90th and 95th percentile. The left-most upper panel in **Figure 5**, for example, considers industries that were at the 10th percentile of the cross-section distribution of comparative advantage in the start year; panel **Figure 5c** shows industries that were at the median of the distribution in the start year. Each curve in a panel then plots the conditional CDF for the transitions from the given percentile in the start year to any percentile of the cross section one year later. By design, data that are re-sampled under an iid distribution would show up at a 45-degree line, while complete persistence of comparative advantage would make the CDF a step function. To characterize the data, we use three windows of annual transitions: the mean annual transitions during the years 1964-67 around the beginning of our sample period, the mean annual transitions during the years 1984-87 around the middle of our sample, and the mean annual transitions during the years 2004-07 towards the end of the sample. These transitions are shown in gray. Our GLD estimation constrains parameters to be constant over time, so the model predicted transition probabilities give rise to a time-invariant CDF shown in blue.

The five-year GLD performs well in capturing the annual dynamics of comparative advantage for most industries. As **Figure 5** shows, the model-predicted conditional CDF's tightly fit their empirical counterparts for industries at the median and higher percentiles in the start year. It is only in the lower tail, in particular around

[43] The Supplementary Material (Table S3) presents GMM results for moments on ten-year intervals using PPML gravity estimates of comparative advantage and the Balassa RCA index.

[44] In the limit when σ becomes arbitrarily large, the GLD would exhibit no persistence, converging to an iid process.

Figure 5: **Diffusion Predicted Annual Transitions**

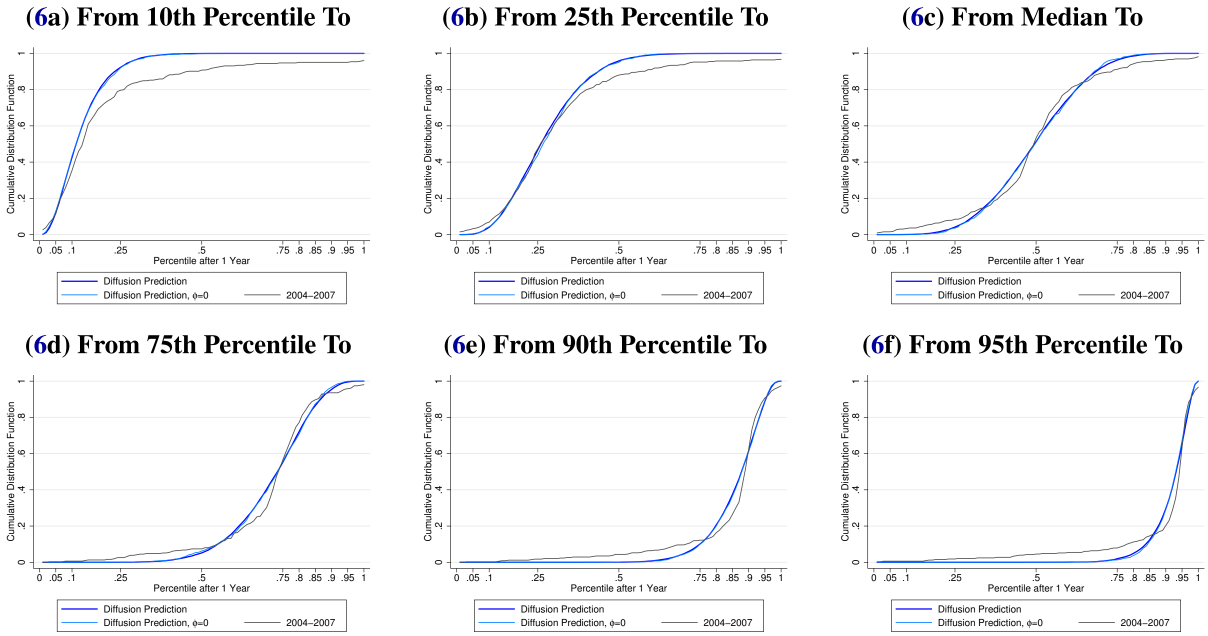


Source: WTF (Feenstra et al. 2005, updated through 2008) for 133 time-consistent industries in 90 countries from 1962-2007 and CEPII.org; OLS gravity measures of export capability (log absolute advantage) $k = \ln A$ from (6).
Note: Predicted cumulative distribution function of comparative advantage $\hat{A}_{is,t+1}$ after one year, given the percentile (10th, 25th, median, 75th, 90th, 95th) of current comparative advantage $\hat{A}_{is,t}$. Predictions based on simulations using estimates from Table 2 (column 1). Observed cumulative distribution function from mean annual transitions during the periods 1964-1967, 1984-1987, and 2004-2007.

the 10th percentile, that the fit of the GLD model becomes less close, though the model predictions are more comparable to the data in later than in earlier periods. Country-industries in the bottom tail have low trade volumes, especially in the early sample period, meaning that estimates of the empirical transition probabilities in the lower tail are not necessarily precisely estimated and may fluctuate more over time. **Figure 5** indicates that the dynamic fit becomes relatively close for percentiles at around the 25th percentile. The discrepancies in the lowest tail notwithstanding, for industries with moderate to high trade values, which account for the bulk of global trade, the model succeeds in matching empirical transition probabilities.

The dynamic transition probabilities implied by the GLD also allow us to assess how well a simple Ornstein-Uhlenbeck process approximates trade dynamics. In a statistical horse race between the unconstrained GLD and the OU process, the former wins—at least for OLS gravity estimates of comparative advantage—because we reject that $\phi = 0$ in **Table 3**, columns 1 to 3. Yet, estimating the unconstrained GLD is substantially more burdensome than estimating the simple discretized linear form of the OU process. For both empirical and theoretical modeling, it is important to understand how much is lost by imposing the constraint that comparative advantage has a stationary log normal distribution.

Figure 6: **Diffusion Predicted Annual Transitions, Constrained and Unconstrained ϕ**



Source: WTF (Feenstra et al. 2005, updated through 2008) for 133 time-consistent industries in 90 countries from 1962-2007; OLS gravity measures of export capability (log absolute advantage) $k = \ln A$ from (6).
Note: Predicted cumulative distribution function of comparative advantage $\hat{A}_{is,t+1}$ after one year, given the percentile (10th, 25th, median, 75th, 90th, 95th) of current comparative advantage $\hat{A}_{is,t}$. Predictions based on simulations using estimates from Table 2 (column 1) and Table 3 (column 2, $\phi = 0$). Observed cumulative distribution function from mean annual transitions during the period 2006-2009.

Following **Figure 5**, we simulate trajectories of the GLD, once from estimates with ϕ unconstrained and once from estimates with $\phi = 0$, using coefficients from columns 1 and 2 in **Table 3**. The simulations predict the theoretical transition probabilities over the one-year horizon across percentiles of the cross-section distribution. **Figure 6** shows the empirical cumulative distribution functions for annual transitions of comparative advantage over the full sample period 1962-2007 (in gray) and compares the empirical distribution to the two model-predicted cumulative distribution functions (light and dark blue), where the fit of the unconstrained GLD model (dark blue) is the same as depicted in **Figure 5** above. As in **Figure 5**, each panel in **Figure 6** considers industries that were at a given percentile of the cross-section distribution of comparative advantage in the start year. Each curve in a panel shows the conditional CDF for the transitions from the given percentile in the start year to any percentile of the cross section one year later. For all start-year percentiles, the model-predicted transitions hardly differ between the constrained specification (light blue) and the unconstrained specification (dark blue). When alternating between the two models, the shapes of the model-predicted conditional CDF's are very similar, even in the upper tail. In the lower tail, where the GLD produces the least tight dynamic fit, the constrained OU specification performs no worse than the unconstrained GLD. The simple OU process thus appears to approximate the empirical dynamics of trade in a manner that is close to indistinguishable from the GLD extension.

5.3 Model fit II: Matching the empirical cross-section distribution

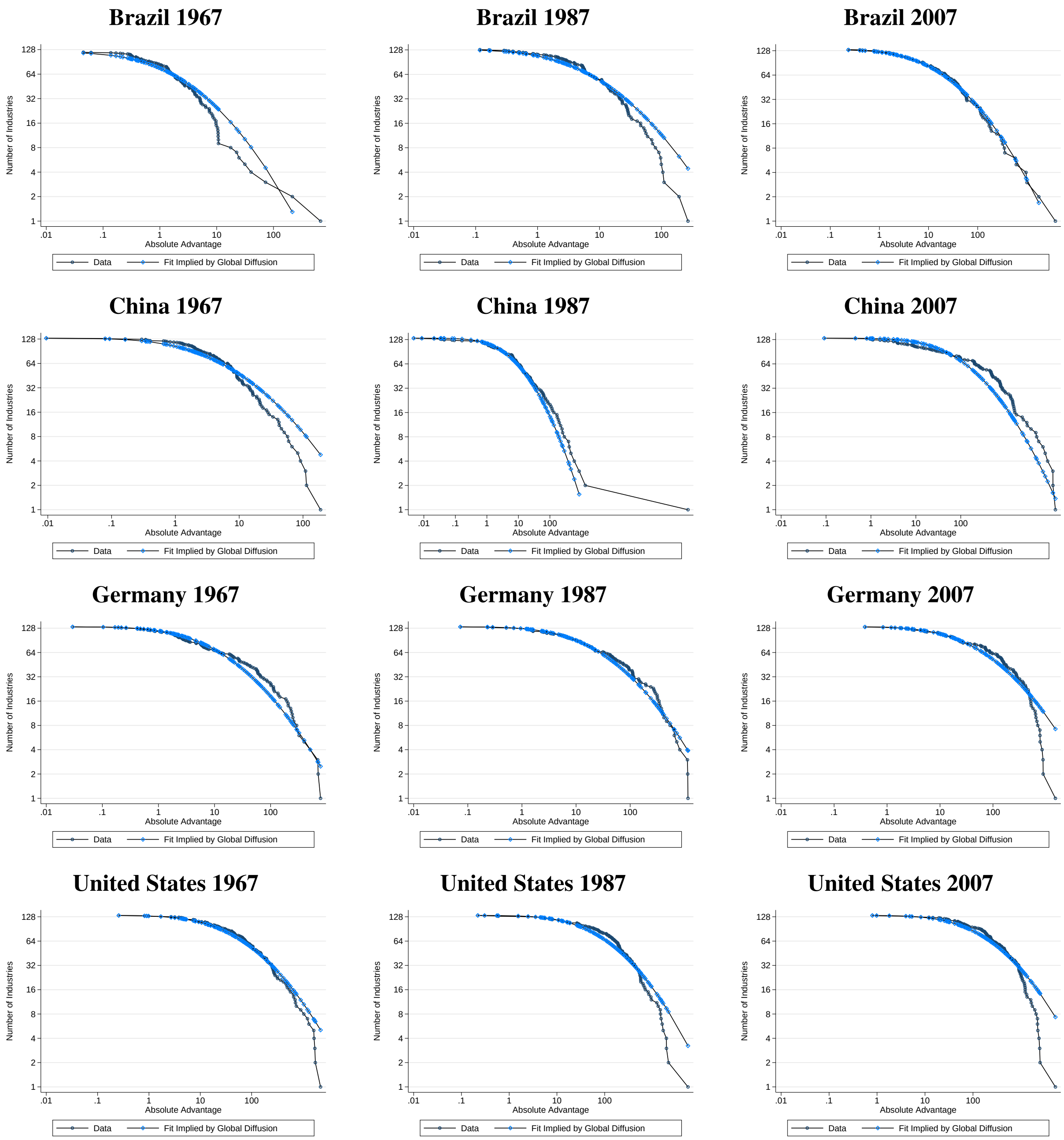
As a closing exercise, we evaluate the fit of our diffusion for comparative advantage by examining how well the GMM parameter estimates describe the cross-sectional distribution of comparative advantage. We have given the GMM estimator a heavy burden: to fit the export dynamics across 90 countries for 46 years using only three time-invariant parameters (η, σ, ϕ), conditional on stochastic country-wide growth trends. Because the moments we use in GMM estimation reflect the time-series behavior of country-industry exports, our estimator fits the diffusion of comparative advantage but not its stationary cross-sectional distribution. We can therefore use the stationary generalized gamma distribution implied by the GLD process to assess how well our model captures the heavy tails of export advantage observed in the repeated cross-section data. For this comparison, we use the benchmark estimates from **Table 2** in column 1. We obtain similar results when we use results for ϕ constrained to zero in column 2 of **Table 3**.

For each country in each year, we project the cross-sectional distribution of comparative advantage implied by the parameters estimated from the diffusion and compare it to the empirical distribution. To implement this validation exercise, we need a measure of $\hat{A}_{ist}$ in (14), the value of which depends on the unobserved country-specific stochastic trend Z_{st}. This trend accounts for the observed horizontal shifts in distribution of log absolute advantage over time, which may result from country-wide technological progress, factor accumulation, or other sources of aggregate growth. In the estimation, we concentrate out Z_{st} by (18), which allows us to estimate its realization for each country in each year. Combining observed absolute advantage A_{ist} with the stochastic-trend estimate allows us to compute realized values of comparative advantage $\hat{A}_{ist}$.

To gauge the goodness of fit of our specification, we first plot our empirical measure of absolute advantage A_{ist}. To do so, following the earlier exercise in **Figure 2**, we rank order the data and plot for each country-industry observation the level of absolute advantage (in log units) against the log number of industries with absolute advantage greater than this value, which is equal to the log of one minus the empirical CDF. To obtain the simulated distribution resulting from the parameter estimates, we plot the global diffusion's implied stationary distribution for the same series. The diffusion implied values are constructed, for each level of A_{ist}, by evaluating the log of one minus the predicted generalized gamma CDF at $\hat{A}_{ist} = A_{ist}/Z_{st}$. The implied distribution thus uses the global diffusion parameter estimates (to project the scale and shape of the CDF) as well as the identified country-specific trend Z_{st} (to project the position of the CDF).

Figure 7 compares plots of the actual data against the GLD-implied distributions for four countries in three

Figure 7: **Diffusion Predicted and Observed Cumulative Probability Distributions of Absolute Advantage for Select Countries in 1967, 1987 and 2007**



Source: WTF (Feenstra et al. 2005, updated through 2008) for 133 time-consistent industries in 90 countries from 1962-2007 and CEPII.org; OLS gravity measures of export capability (log absolute advantage) $k = \ln A$ from (6).
Note: The graphs show the observed and the predicted frequency of industries (the cumulative probability $1 - F_A(a)$ times the total number of industries $I = 133$) on the vertical axis plotted against the level of absolute advantage a (such that $A_{ist} \geq a$) on the horizontal axis. Both axes have a log scale. The predicted frequencies are based on the GMM estimates of the comparative advantage diffusion (17) in Table 2 (parameters η and ϕ in column 1) and the inferred country-specific stochastic trend component $\ln Z_{st}$ from (18), which horizontally shifts the distributions but does not affect their shape.

years, 1967, 1987, 2007. **Figures A4, A5** and **A6** in the Appendix present plots in these years for the 28 countries that are also shown in **Figures A1, A2** and **A3**.[45] While **Figures A1** through **A3** depicted Pareto and log normal maximum likelihood estimates of each individual country's cross-sectional distribution by year (number of parameters estimated = number of countries × number of years), our exercise now is vastly more parsimonious and based on a fit of the time-series evolution, not the observed cross sections. **Figure 7** and Appendix **Figures A4** through **A6** show that the empirical distributions and the GLD-implied distributions have the same concave shape and horizontally shifting position. Considering that the shape of the distribution effectively depends on only two parameters for all country-industries and years, the GLD-predicted distributions are remarkably accurate. There are important differences between the actual and predicted plots in only a few countries and a few years, including China in 1987, Russia in 1987 and 2007, Taiwan in 1987, and Vietnam in 1987 and 2007. Three of these four cases involve countries undergoing a transition away from central planning during the designated time period, suggesting periods of economic disruption.

There are other, minor discrepancies between the empirical distributions and the GLD-implied distributions that merit further attention. In 2007 in a handful of countries in East and Southeast Asia—China, Japan, Rep. Korea, Malaysia, Taiwan, and Vietnam—the empirical distributions exhibit less concavity than the generalized gamma distributions (or the log normal for that matter). These countries show more mass in the upper tail of comparative advantage than they ought, implying that they excel in too many sectors, relative to the norm. It remains to be investigated whether these differences in fit are associated with conditions in the countries themselves or with the particular industries in which these countries tend to specialize.

The noticeable deviations for some countries in certain years notwithstanding, across countries and for the full sample period the percentiles of the country-level distributions of comparative advantage are remarkably stable for each of our three measures of comparative advantage. This stability suggests that there is a unifying global and stationary distribution of comparative advantage. Our estimates of the GLD time series imply shape-parameter values of a generalized gamma CDF, and those predicted shape parameters tightly fit the relevant percentiles of the global comparative-advantage distribution.[46]

6 Conclusion

The traditional Ricardian trade model has long presented a conundrum to economists. Although it offers a simple and intuitive characterization of comparative advantage, it yields knife-edge predictions for country specialization

[45] Because the country-specific trend Z_{st} shifts the implied stationary distribution horizontally, we clarify fit by cutting the depicted part of that single distribution at the lower and upper bounds of the specific country's observed support in a given year.

[46] The Supplementary Material (Figure S11) shows percentile plots for OLS- and PPML-based measures of export capability over time and the fit of our according GLD estimates to those percentiles.

patterns that fit the data poorly. Eaton and Kortum (2002) have reinvigorated the Ricardian framework. By treating the capability of firms from a country in a sector as probabilistic rather than deterministic, they derive realistically complex country specialization patterns and provide a robust framework for quantitative analysis. The primitives in the EK model are the parameters of the distribution for industry productivity, which pin down country export capabilities. Comparative advantage arises from these capabilities varying across countries. Our goal in this paper is to characterize the dynamic empirical properties of export capability in order to inform modeling of the deep origins of comparative advantage.

The starting point for our analysis is two strong empirical regularities in trade that economists have studied incompletely and in isolation. Many papers have noted the tendency for countries to concentrate their exports in a relatively small number of sectors. Our first contribution is to show that this concentration arises from a heavy-tailed distribution of industry export capability that is approximately log normal and whose shape is stable across countries, sectors, and time. Likewise, the trade literature has detected in various forms a tendency for mean reversion in national industry productivities. Our second contribution is to establish that mean reversion in export capability, rather than indicative of convergence in productivities and degeneracy in comparative advantage, is instead consistent with a stationary stochastic process, whose properties are common across borders and sectors. In literatures on the growth of cities and the growth of firms, economists have used stochastic processes to study the determinants of the long-run distribution of sizes. Our third contribution is to develop an analogous empirical framework for identifying the parameters that govern the stationary distribution of export capability. The main result of this analysis is that log normality offers a reasonable approximation. The stochastic process that generates log normality can be estimated in its discretized form by simple linear regression.

In the stochastic process that we estimate, country export capabilities evolve independently across industries, subject to controls for aggregate country growth, and independently across countries, subject to controls for global industry growth. This approach runs counter to recent theoretical research in trade, which examines how innovations to productivity are transmitted across space and time. Our analysis can be extended straightforwardly to allow for such interactions. The Ornstein-Uhlenbeck process generalizes to a multivariate diffusion, in which stochastic innovations to an industry in one country also affect related industries in the same economy (or the same industry in a nation's trading partners). Because of the linearity of the discretized OU process, it is feasible to estimate such interactions while still identifying the parameters that characterize the stationary distribution of comparative advantage. An obvious next step in the analysis is to model diffusions that allow for such intersectoral and international productivity linkages.

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Appendix

A Generalized Logistic Diffusion

The principal insights of Subsections 4.1 and 4.2 are based on the following relationship.

Lemma 1. *The generalized logistic diffusion*

$$\frac{\mathrm{d}\hat{A}_{is}(t)}{\hat{A}_{is}(t)} = \frac{\sigma^2}{2}\left[1 - \eta\,\frac{\hat{A}_{is}(t)^{\phi} - 1}{\phi}\right]\mathrm{d}t + \sigma\,\mathrm{d}W_{is}^{\hat{A}}(t) \tag{A.1}$$

for real parameters (η, σ, ϕ) *has a stationary distribution that is generalized gamma with a probability density* $f_{\hat{A}}(\hat{a}_{is}|\hat{\theta}, \kappa, \phi)$ *given by* (15) *and the real parameters*

$$\hat{\theta} = \left(\phi^2/\eta\right)^{1/\phi} > 0 \quad \textit{and} \quad \kappa = 1/\hat{\theta}^{\phi} > 0.$$

A non-degenerate stationary distribution exists only if $\eta > 0$.

Equation (A.1) restates equation (16) from the text.

A.1 Derivation of the generalized logistic diffusion

We now establish Lemma 1. As a starting point, note that the ordinary gamma distribution is known to be the stationary distribution of the stochastic logistic equation (Leigh 1968). We generalize this ordinary logistic diffusion to yield a generalized gamma distribution as the stationary distribution in the cross section. Our (three-parameter) generalization of the gamma distribution relates back to the ordinary (two-parameter) gamma distribution through a power transformation. Take an ordinary gamma distributed random variable X with two parameters $\bar{\theta}, \kappa > 0$ and the density function

$$f_X(x|\bar{\theta}, \kappa) = \frac{1}{\Gamma(\kappa)}\frac{1}{\bar{\theta}}\left(\frac{x}{\bar{\theta}}\right)^{\kappa-1}\exp\left\{-\frac{x}{\bar{\theta}}\right\} \qquad \text{for} \quad x > 0. \tag{A.2}$$

Then the transformed variable $\hat{A} = X^{1/\phi}$ has a generalized gamma distribution under the accompanying parameter transformation $\hat{\theta} = \bar{\theta}^{1/\phi}$ because

$$\begin{aligned} f_{\hat{A}}(\hat{a}|\hat{\theta}, \kappa, \phi) &= \tfrac{\partial}{\partial \hat{a}}\Pr(\hat{A} \leq \hat{a}) = \tfrac{\partial}{\partial \hat{a}}\Pr(X^{1/\phi} \leq \hat{a}) \\ &= \tfrac{\partial}{\partial \hat{a}}\Pr(X \leq \hat{a}^{\phi}) = f_X(\hat{a}^{\phi}|\hat{\theta}^{\phi}, \kappa)\cdot|\phi\hat{a}^{\phi-1}| \\ &= \frac{\hat{a}^{\phi-1}}{\Gamma(\kappa)}\left|\frac{\phi}{\hat{\theta}^{\phi}}\right|\left(\frac{\hat{a}^{\phi}}{\hat{\theta}^{\phi}}\right)^{\kappa-1}\exp\left\{-\frac{\hat{a}^{\phi}}{\hat{\theta}^{\phi}}\right\} = \frac{1}{\Gamma(\kappa)}\left|\frac{\phi}{\hat{\theta}}\right|\left(\frac{\hat{a}}{\hat{\theta}}\right)^{\phi\kappa-1}\exp\left\{-\left(\frac{\hat{a}}{\hat{\theta}}\right)^{\phi}\right\}, \end{aligned}$$

which is equivalent to the generalized gamma probability density function (15), where $\Gamma(\cdot)$ denotes the gamma function and $\hat{\theta}, \kappa, \phi$ are the three parameters of the generalized gamma distribution in our context ($\hat{a} > 0$ can be arbitrarily close to zero).

The ordinary logistic diffusion of a variable X follows the stochastic process

$$\mathrm{d}X(t) = \left[\bar{\alpha} - \bar{\beta}\,X(t)\right]X(t)\,\mathrm{d}t + \bar{\sigma}\,X(t)\,\mathrm{d}W(t) \qquad \text{for} \quad X(t) > 0, \tag{A.3}$$

where $\bar{\alpha}, \bar{\beta}, \bar{\sigma} > 0$ are parameters, t denotes time, $W(t)$ is the Wiener process (standard Brownian motion) and a reflection ensures that $X(t) > 0$. The stationary distribution of this process (the limiting distribution of $X = X(\infty) = \lim_{t\to\infty} X(t)$) is known to be an ordinary gamma distribution (Leigh 1968):

$$f_X(x|\bar{\theta}, \kappa) = \frac{1}{\Gamma(\kappa)} \left|\frac{1}{\bar{\theta}}\right| \left(\frac{x}{\bar{\theta}}\right)^{\kappa-1} \exp\left\{-\frac{x}{\bar{\theta}}\right\} \qquad \text{for} \quad x > 0, \tag{A.4}$$

as in (A.2) with

$$\begin{aligned} \bar{\theta} &= \bar{\sigma}^2/(2\bar{\beta}) > 0, \\ \kappa &= 2\bar{\alpha}/\bar{\sigma}^2 - 1 > 0 \end{aligned} \tag{A.5}$$

under the restriction $\bar{\alpha} > \bar{\sigma}^2/2$. The ordinary logistic diffusion can also be expressed in terms of infinitesimal parameters as

$$\mathrm{d}X(t) = \mu_X(X(t))\,\mathrm{d}t + \sigma_X(X(t))\,\mathrm{d}W(t) \qquad \text{for} \quad X(t) > 0,$$
$$\mu_X(X) = (\bar{\alpha} - \bar{\beta}\,X)X \quad \text{and} \quad \sigma_X^2(X) = \bar{\sigma}^2\,X^2.$$

Now consider the diffusion of the transformed variable $\hat{A}(t) = X(t)^{1/\phi}$. In general, a strictly monotone transformation $\hat{A} = g(X)$ of a diffusion X is a diffusion with infinitesimal parameters

$$\mu_{\hat{A}}(\hat{A}) = \frac{1}{2}\sigma_X^2(X)g''(X) + \mu_X(X)g'(X) \quad \text{and} \quad \sigma_{\hat{A}}^2(\hat{A}) = \sigma_X^2(X)g'(X)^2$$

(see Karlin and Taylor 1981, Section 15.2, Theorem 2.1). Applying this general result to the specific monotone transformation $\hat{A} = X^{1/\phi}$ yields our specification of a *generalized logistic diffusion*:

$$\mathrm{d}\hat{A}(t) = \left[\alpha - \beta\hat{A}(t)^{\phi}\right]\hat{A}(t)\,\mathrm{d}t + \sigma\hat{A}(t)\,\mathrm{d}W(t) \qquad \text{for} \quad \hat{A}(t) > 0. \tag{A.6}$$

with the parameters

$$\alpha \equiv \left[\frac{1-\phi}{2}\frac{\bar{\sigma}^2}{\phi^2} + \frac{\bar{\alpha}}{\phi}\right], \qquad \beta \equiv \frac{\bar{\beta}}{\phi}, \qquad \sigma \equiv \frac{\bar{\sigma}}{\phi}. \tag{A.7}$$

The term $-\beta\hat{A}(t)^{\phi}$ now involves a power function and the parameters of the generalized logistic diffusion collapse to the parameters of the ordinary logistic diffusion for $\phi = 1$.

We infer that the stationary distribution of $\hat{A}(\infty) = \lim_{t\to\infty}\hat{A}(t)$ is a generalized gamma distribution by (15) and by the derivations above:

$$f_{\hat{A}}(\hat{a}|\hat{\theta}, \kappa, \phi) = \frac{1}{\Gamma(\kappa)} \left|\frac{\phi}{\hat{\theta}}\right| \left(\frac{\hat{a}}{\hat{\theta}}\right)^{\phi\kappa-1} \exp\left\{-\left(\frac{\hat{a}}{\hat{\theta}}\right)^{\phi}\right\} \qquad \text{for} \quad x > 0,$$

with

$$\begin{aligned} \hat{\theta} &= \bar{\theta}^{1/\phi} = [\bar{\sigma}^2/(2\bar{\beta})]^{1/\phi} = [\phi\sigma^2/(2\beta)]^{1/\phi} > 0, \\ \kappa &= 2\bar{\alpha}/\bar{\sigma}^2 - 1 = [2\alpha/\sigma^2 - 1]/\phi > 0 \end{aligned} \tag{A.8}$$

by (A.5) and (A.7).

A.2 Existence and parametrization

Existence of a non-degenerate stationary distribution with $\hat{\theta}, \kappa > 0$ circumscribes how the parameters of the diffusion α, β, σ and ϕ must relate to each other. A strictly positive $\hat{\theta}$ implies that $\text{sign}(\beta) = \text{sign}(\phi)$. Second, a strictly positive κ implies that $\text{sign}(\alpha - \sigma^2/2) = \text{sign}(\phi)$. The latter condition is closely related to the requirement that comparative advantage neither collapse nor explode. If the level elasticity of dissipation ϕ is strictly positive ($\phi > 0$) then, for the stationary probability density $f_{\hat{A}}(\cdot)$ to be non-degenerate, the offsetting constant drift parameter α needs to strictly exceed the variance of the stochastic innovations: $\alpha \in (\sigma^2/2, \infty)$. Otherwise absolute advantage would "collapse" as arbitrarily much time passes, implying industries die out. If $\phi < 0$ then the offsetting positive drift parameter α needs to be strictly less than the variance of the stochastic innovations: $\alpha \in (-\infty, \sigma^2/2)$; otherwise absolute advantage would explode.

Our preferred parametrization of the generalized logistic diffusion is (A.1) in Lemma 1 for real parameters η, σ, ϕ. That parametrization can be related back to the parameters in (A.6) by setting $\alpha = (\sigma^2/2) + \beta$ and $\beta = \eta\sigma^2/(2\phi)$. In this simplified formulation, the no-collapse and no-explosion conditions are satisfied for the single restriction that $\eta > 0$. The reformulation in (A.1) also clarifies that one can view our generalization of the drift term $[\hat{A}_{is}(t)^\phi - 1]/\phi$ as a conventional Box-Cox transformation of $\hat{A}_{is}(t)$ to model the level dependence.

The non-degenerate stationary distribution accommodates both the log normal and the Pareto distribution as limiting cases. When $\phi \to 0$, both α and β tend to infinity; if β did not tend to infinity, a drifting random walk would result in the limit. A stationary log normal distribution requires that $\alpha/\beta \to 1$, so $\alpha \to \infty$ at the same rate with $\beta \to \infty$ as $\phi \to 0$. For existence of a non-degenerate stationary distribution, in the benchmark case with $\phi \to 0$ we need $1/\alpha \to 0$ for the limiting distribution to be log normal. In contrast, a stationary Pareto distribution with shape parameter p would require that $\alpha = (2-p)\sigma^2/2$ as $\phi \to 0$ (see e.g. Crooks 2010, Table 1; proofs are also available from the authors upon request).

A.3 From comparative to absolute advantage

If comparative advantage $\hat{A}_{is}(t)$ follows a generalized logistic diffusion by (A.1), then the stationary distribution of comparative advantage is a generalized gamma distribution with density (15) and parameters $\hat{\theta} = \left(\phi^2/\eta\right)^{1/\phi} > 0$ and $\kappa = 1/\hat{\theta}^\phi > 0$ by Lemma 1. From this stationary distribution of comparative advantage $\hat{A}_{is}$, we can infer the cross-sectional distribution of absolute advantage $A_{is}(t)$. Note that, by definition (14), absolute advantage is not necessarily stationary because the stochastic trend $Z_s(t)$ may not be stationary.

Absolute advantage is related to comparative advantage through a country-wide stochastic trend by definition (14). Plugging this definition into (15), we can infer that the probability density of absolute advantage must be proportional to

$$f_A(a_{is}|\hat{\theta}, Z_s(t), \kappa, \phi) \propto \left(\frac{a_{is}}{\hat{\theta} Z_s(t)}\right)^{\phi\kappa - 1} \exp\left\{-\left(\frac{a_{is}}{\hat{\theta} Z_s(t)}\right)^\phi\right\}.$$

It follows from this proportionality that the probability density of absolute advantage must be a generalized gamma distribution with $\theta_s(t) = \hat{\theta} Z_s(t) > 0$, which is time varying because of the stochastic trend $Z_s(t)$. We summarize these results in a lemma.

Lemma 2. *If comparative advantage $\hat{A}_{is}(t)$ follows a generalized logistic diffusion* (A.1) *with real parameters η, σ, ϕ ($\eta > 0$), then the cross-sectional distribution of absolute advantage $A_{is}(t)$ is generalized gamma with the CDF*

$$F_A(a_{is}|\theta_s(t), \phi, \kappa) = G\left[\left(\frac{a_{is}}{\theta_s(t)}\right)^\phi; \kappa\right] \tag{A.9}$$

for the strictly positive parameters

$$\hat{\theta} = \left(\phi^2/\eta\right)^{1/\phi}, \quad \theta_s(t) = \hat{\theta} Z_s(t) \quad \textit{and} \quad \kappa = 1/\hat{\theta}^{\phi}.$$

Proof. Derivations above establish that the cross-sectional distribution of absolute advantage is generalized gamma. The cumulative distribution function follows from Kotz et al. (1994, Ch. 17, Section 8.7). □

Lemma 2 establishes that the diffusion and cross-sectional distribution of absolute advantage inherit all relevant properties of comparative advantage after adjustment for an (arbitrary) country-level growth trend. Equation (A.9) predicts cumulative probability distributions of absolute advantage such as those in **Figure 2** (and in Appendix **Figures A1, A2** and **A3**). The lower cutoff for absolute advantage shifts right over time, but the shape of the cross sectional CDF is stable across countries and years. We will document in Appendix B how the trend can be recovered from estimation of the comparative-advantage diffusion using absolute advantage data.

A.4 Moments and the mean-median ratio

As a prelude to the GMM estimation, the r-th raw moments of the ratios $a_{is}/\theta_s(t)$ and $\hat{a}_{is}/\hat{\theta}$ are

$$\mathbb{E}\left[\left(\frac{a_{is}}{\theta_s(t)}\right)^r\right] = \mathbb{E}\left[\left(\frac{\hat{a}_{is}}{\hat{\theta}}\right)^r\right] = \frac{\Gamma(\kappa + r/\phi)}{\Gamma(\kappa)}$$

and identical because both $[a_{is}/\theta_s(t)]^{1/\phi}$ and $[\hat{a}_{is}/\hat{\theta}]^{1/\phi}$ have the same standard gamma distribution (Kotz et al. 1994, Ch. 17, Section 8.7). As a consequence, the raw moments of absolute advantage A_{is} are scaled by a country-specific time-varying factor $Z_s(t)^r$ whereas the raw moments of comparative advantage are constant over time if comparative advantage follows a diffusion with three constant parameters $(\hat{\theta}, \kappa, \phi)$:

$$\mathbb{E}\left[(a_{is})^r \middle| Z_s(t)^r\right] = Z_s(t)^r \cdot \mathbb{E}\left[(\hat{a}_{is})^r\right] = Z_s(t)^r \cdot \hat{\theta}^r \frac{\Gamma(\kappa + r/\phi)}{\Gamma(\kappa)}.$$

By Lemma 2, the median of comparative advantage is $\hat{a}_{.5} = \hat{\theta}(G^{-1}[.5;\kappa])^{1/\phi}$. A measure of concentration in the right tail is the ratio of the mean and the median, which is independent of $\hat{\theta}$ and equals

$$\textit{Mean/median ratio} = \frac{\Gamma(\kappa + 1/\phi)/\Gamma(\kappa)}{(G^{-1}[.5;\kappa])^{1/\phi}}. \tag{A.10}$$

We report this measure of concentration to characterize the curvature of the stationary distribution.

B Identification of the Generalized Logistic Diffusion

Our implementation of the Generalized Logistic Diffusion requires not only identification of the three time-invariant real parameters (η, σ, ϕ)—or equivalently $(\hat{\theta}, \kappa, \phi)$—, but also identification of a stochastic trend: the country-specific time-varying factor $Z_s(t)$.

Proposition 1. *If comparative advantage $\hat{A}_{is}(t)$ follows the generalized logistic diffusion (A.1) with real parameters η, σ, ϕ ($\eta > 0$), then the country specific stochastic trend $Z_s(t)$ is recovered from the first moment of the logarithm of absolute advantage as:*

$$Z_s(t) = \exp\left\{\mathbb{E}_{st}[\ln A_{is}(t)] - \frac{\ln(\phi^2/\eta) + \Gamma'(\eta/\phi^2)/\Gamma(\eta/\phi^2)}{\phi}\right\} \tag{B.11}$$

where $\Gamma'(\kappa)/\Gamma(\kappa)$ is the digamma function.

Equation (B.11) restates equation (18) from the text. For a proof of Proposition 1, first consider a random variable X that has a gamma distribution with scale parameter θ and shape parameter κ. For any power $n \in \mathbb{N}$ we have

$$\begin{aligned}
\mathbb{E}\left[\ln(X^n)\right] &= \int_0^\infty \ln(x^n)\frac{1}{\Gamma(\kappa)}\frac{1}{\theta}\left(\frac{x}{\theta}\right)^{\kappa-1}\exp\left\{-\frac{x}{\theta}\right\}\mathrm{d}x \\
&= \frac{n}{\Gamma(\kappa)}\int_0^\infty \ln(\theta z)z^{\kappa-1}e^{-z}\mathrm{d}z \\
&= n\ln\theta + \frac{n}{\Gamma(\kappa)}\int_0^\infty \ln(z)z^{\kappa-1}e^{-z}\mathrm{d}z \\
&= n\ln\theta + \frac{n}{\Gamma(\kappa)}\frac{\partial}{\partial\kappa}\int_0^\infty z^{\kappa-1}e^{-z}\mathrm{d}z \\
&= n\ln\theta + n\frac{\Gamma'(\kappa)}{\Gamma(\kappa)},
\end{aligned}$$

where $\Gamma'(\kappa)/\Gamma(\kappa)$ is the digamma function.

From Appendix A (Lemma 1) we know that raising a gamma random variable to the power $1/\phi$ creates a generalized gamma random variable $X^{1/\phi}$ with shape parameters κ and ϕ and scale parameter $\theta^{1/\phi}$. Therefore

$$\mathbb{E}\left[\ln(X^{1/\phi})\right] = \frac{1}{\phi}\mathbb{E}\left[\ln X\right] = \frac{\ln(\theta) + \Gamma'(\kappa)/\Gamma(\kappa)}{\phi}$$

This result allows us to identify the country specific stochastic trend $X_s(t)$.

For $\hat{A}_{is}(t)$ has a generalized gamma distribution across i for any given s and t with shape parameters ϕ and η/ϕ^2 and scale parameter $(\phi^2/\eta)^{1/\phi}$ we have

$$\mathbb{E}_{st}\left[\ln \hat{A}_{is}(t)\right] = \frac{\ln(\phi^2/\eta) + \Gamma'(\eta/\phi^2)/\Gamma(\eta/\phi^2)}{\phi}.$$

From definition (14) and $\hat{A}_{is}(t) = A_{is}(t)/Z_s(t)$ we can infer that $\mathbb{E}_{st}[\ln \hat{A}_{is}(t)] = \mathbb{E}_{st}[\ln A_{is}(t)] - \ln Z_s(t)$. Re-arranging and using the previous result for $\mathbb{E}[\ln \hat{A}_{is}(t) \mid s, t]$ yields

$$Z_s(t) = \exp\left\{\mathbb{E}_{st}[\ln A_{is}(t)] - \frac{\ln(\phi^2/\eta) + \Gamma'(\eta/\phi^2)/\Gamma(\eta/\phi^2)}{\phi}\right\}$$

as stated in the text.

C GMM Estimation of the Associated Pearson-Wong Process

GMM estimation of the Generalized Logistic Diffusion requires conditional moments, which we obtain from a Pearson-Wong transformation.

Proposition 2. *If comparative advantage $\hat{A}_{is}(t)$ follows the generalized logistic diffusion (A.1) with real parameters η, σ, ϕ ($\eta > 0$), then the following two statements are true.*

- *The transformed variable*

$$\hat{B}_{is}(t) = [\hat{A}_{is}(t)^{-\phi} - 1]/\phi \qquad \text{(C.12)}$$

follows the diffusion

$$\mathrm{d}\hat{B}_{is}(t) = -\frac{\sigma^2}{2}\left[\left(\eta - \phi^2\right)\hat{B}_{is}(t) - \phi\right]\mathrm{d}t + \sigma\sqrt{\phi^2\hat{B}_{is}(t)^2 + 2\phi\hat{B}_{is}(t) + 1}\,\mathrm{d}W_{is}^{\hat{B}}(t)$$

and belongs to the Pearson-Wong family.

- *For any time t, time interval $\Delta > 0$, and integer $n \leq M < \eta/\phi^2$, the n-th conditional moment of the transformed process $\hat{B}_{is}(t)$ satisfies the recursive condition:*

$$\mathbb{E}\left[\hat{B}_{is}(t+\Delta)^n \,\middle|\, \hat{B}_{is}(t) = b\right] = \exp\left\{-a_n\Delta\right\}\sum_{m=0}^{n}\pi_{n,m}b^m - \sum_{m=0}^{n-1}\pi_{n,m}\mathbb{E}\left[\hat{B}_{is}(t+\Delta)^m \,\middle|\, \hat{B}_{is}(t) = b\right], \tag{C.13}$$

for coefficients a_n and $\pi_{n,m}$ ($n, m = 1, \ldots, M$) as defined below.

Equation (C.12) restates equation (20) in the text.

C.1 Derivation of the Pearson-Wong transform

To establish Proposition 2, first consider a random variable X with a standard logistic diffusion (the $\phi = 1$ case). The Bernoulli transformation $1/X$ maps the standard logistic diffusion into the Pearson-Wong family (see e.g. Prajneshu 1980, Dennis 1989). Similar to our derivation of the generalized logistic diffusion in Appendix A, we follow up on that transformation with an additional Box-Cox transformation and apply $\hat{B}_{is}(t) = [\hat{A}_{is}(t)^{-\phi} - 1]/\phi$ to comparative advantage, as stated in (C.12). Define $W_{is}^{\hat{B}}(t) \equiv -W_{is}^{\hat{A}}(t)$. Then $\hat{A}_{is}^{-\phi} = \phi\hat{B}_{is}(t) + 1$ and, by Itō's lemma,

$$\begin{aligned}
\mathrm{d}\hat{B}_{is}(t) &= \mathrm{d}\left(\frac{\hat{A}_{is}(t)^{-\phi} - 1}{\phi}\right) \\
&= -\hat{A}_{is}(t)^{-\phi-1}\,\mathrm{d}\hat{A}_{is}(t) + \frac{1}{2}(\phi+1)\hat{A}_{is}(t)^{-\phi-2}(\mathrm{d}\hat{A}_{is}(t))^2 \\
&= -\hat{A}_{is}(t)^{-\phi-1}\left[\frac{\sigma^2}{2}\left(1 - \eta\,\frac{\hat{A}_{is}(t)^{\phi} - 1}{\phi}\right)\hat{A}_{is}(t)\,\mathrm{d}t + \sigma\hat{A}_{is}(t)\,\mathrm{d}W_{is}^{\hat{A}}(t)\right] \\
&\qquad + \frac{1}{2}(\phi+1)\hat{A}_{is}(t)^{-\phi-2}\sigma^2\hat{A}_{is}(t)^2\,\mathrm{d}t \\
&= -\frac{\sigma^2}{2}\left[\left(1 + \frac{\eta}{\phi}\right)\hat{A}_{is}(t)^{-\phi} - \frac{\eta}{\phi}\right]\mathrm{d}t - \sigma\hat{A}_{is}(t)^{-\phi}\,\mathrm{d}W_{is}^{\hat{A}}(t) + \frac{\sigma^2}{2}(\phi+1)\hat{A}_{is}(t)^{-\phi}\,\mathrm{d}t \\
&= -\frac{\sigma^2}{2}\left[\left(\frac{\eta}{\phi} - \phi\right)\hat{A}_{is}(t)^{-\phi} - \frac{\eta}{\phi}\right]\mathrm{d}t - \sigma\hat{A}_{is}(t)^{-\phi}\,\mathrm{d}W_{is}^{\hat{A}}(t) \\
&= -\frac{\sigma^2}{2}\left[\left(\frac{\eta}{\phi} - \phi\right)(\phi\hat{B}_{is}(t) + 1) - \frac{\eta}{\phi}\right]\mathrm{d}t + \sigma(\phi\hat{B}_{is}(t) + 1)\,\mathrm{d}W_{is}^{\hat{B}}(t) \\
&= -\frac{\sigma^2}{2}\left[\left(\eta - \phi^2\right)\hat{B}_{is}(t) - \phi\right]\mathrm{d}t + \sigma\sqrt{\phi^2\hat{B}_{is}(t)^2 + 2\phi\hat{B}_{is}(t) + 1}\,\mathrm{d}W_{is}^{\hat{B}}(t).
\end{aligned}$$

The mirror diffusion $\hat{B}_{is}(t)$ is therefore a Pearson-Wong diffusion of the form:

$$\mathrm{d}\hat{B}_{is}(t) = -q(\hat{B}_{is}(t) - \bar{B})\,\mathrm{d}t + \sqrt{2q(a\hat{B}_{is}(t)^2 + b\hat{B}_{is}(t) + c)}\,\mathrm{d}W_{is}^{\hat{B}}(t),$$

where $q = (\eta - \phi^2)\sigma^2/2$, $\bar{B} = \sigma^2\phi/(2q)$, $a = \phi^2\sigma^2/(2q)$, $b = \phi\sigma^2/q$, and $c = \sigma^2/(2q)$.

To construct a GMM estimator based on this Pearson-Wong representation, we apply results in Forman and Sørensen (2008) to construct closed form expressions for the conditional moments of the transformed data and then use these moment conditions for estimation. This technique relies on the convenient structure of the Pearson-Wong class and a general result in Kessler and Sørensen (1999) on calculating conditional moments of diffusion processes using the eigenfunctions and eigenvalues of the diffusion's infinitesimal generator.[47]

A Pearson-Wong diffusion's drift term is affine and its dispersion term is quadratic. Its infinitesimal generator must therefore map polynomials to equal or lower order polynomials. As a result, solving for eigenfunctions and eigenvalues amounts to matching coefficients on polynomial terms. This key observation allows us to estimate the mirror diffusion of the generalized logistic diffusion model and to recover the generalized logistic diffusion's parameters.

Given an eigenfunction and eigenvalue pair (h_s, λ_s) of the infinitesimal generator of $\hat{B}_{is}(t)$, we can follow Kessler and Sørensen (1999) and calculate the conditional moment of the eigenfunction:

$$\mathbb{E}\left[\hat{B}_{is}(t+\Delta)\,\middle|\,\hat{B}_{is}(t)\right] = \exp\{\lambda_s t\}\, h(\hat{B}_{is}(t)). \tag{C.14}$$

Since we can solve for polynomial eigenfunctions of the infinitesimal generator of $B_{is}(t)$ by matching coefficients, this results delivers closed form expressions for the conditional moments of the mirror diffusion for $\hat{B}_{is}(t)$.

To construct the coefficients of these eigen-polynomials, it is useful to consider the case of a general Pearson-Wong diffusion $X(t)$. The stochastic differential equation governing the evolution of $X(t)$ must take the form:

$$\mathrm{d}X(t) = -q(X(t) - \bar{X}) + \sqrt{2(aX(t)^2 + bX(t) + c)\Gamma'(\kappa)/\Gamma(\kappa)}\,\mathrm{d}W^X(t).$$

A polynomial $p_n(x) = \sum_{m=0}^{n} \pi_{n,m} x^m$ is an eigenfunction of the infinitesimal generator of this diffusion if there is some associated eigenvalue $\lambda_n \neq 0$ such that

$$-q(x - \bar{X}) \sum_{m=1}^{n} \pi_{n,m} m x^{m-1} + \theta(ax^2 + bx + c) \sum_{m=2}^{n} \pi_{n,m} m(m-1) x^{m-2} = \lambda_n \sum_{m=0}^{n} \pi_{n,m} x^m$$

We now need to match coefficients on terms.

From the x^n term, we must have $\lambda_n = -n[1 - (n-1)a]q$. Next, normalize the polynomials by setting $\pi_{m,m} = 1$ and define $\pi_{m,m+1} = 0$. Then matching coefficients to find the lower order terms amounts to backward recursion from this terminal condition using the equation

$$\pi_{n,m} = \frac{b_{m+1}}{a_m - a_n} \pi_{n,m+1} + \frac{k_{m+2}}{a_m - a_n} \pi_{n,m+2} \tag{C.15}$$

with $a_m \equiv m[1 - (m-1)a]q$, $b_m \equiv m[\bar{X} + (m-1)b]q$, and $c_m \equiv m(m-1)cq$. Focusing on polynomials with order of $n < (1 + 1/a)/2$ is sufficient to ensure that $a_m \neq a_n$ and avoid division by zero.

Using the normalization that $\pi_{n,n} = 1$, equation (C.14) implies a recursive condition for these conditional

[47] For a diffusion

$$\mathrm{d}X(t) = \mu_X(X(t))\,\mathrm{d}t + \sigma_X(X(t))\,\mathrm{d}W^X(t)$$

the infinitesimal generator is the operator on twice continuously differentiable functions f defined by $A(f)(x) = \mu_X(x)\,\mathrm{d}/\mathrm{d}x + \frac{1}{2}\sigma_X(x)^2\,\mathrm{d}^2/\mathrm{d}x^2$. An eigenfunction with associated eigenvalue $\lambda \neq 0$ is any function h in the domain of A satisfying $Ah = \lambda h$.

moments:

$$\mathbb{E}\left[X(t+\Delta)^n\right)|X(t)=x] = \exp\{-a_n\Delta\}\sum_{m=0}^{n}\pi_{n,m}x^m - \sum_{m=0}^{n-1}\pi_{n,m}\mathbb{E}\left[X(t+\Delta)^m\,|X(t)=x\right].$$

These moments exist if we restrict ourselves to the first $N < (1+1/a)/2$ moments.

C.2 Conditional moment recursion

To arrive at the result in the second part of Proposition 2, set the parameters as $q_s = \sigma^2(\eta-\phi^2)/2$, $\bar{X}_s = \phi/(\eta-\phi^2)$, $a_s = \phi^2/(\eta-\phi^2)$, $b_s = 2\phi/(\eta-\phi^2)$, and $c_s = 1/(\eta-\phi^2)$. From these parameters, we can construct eigenvalues and their associated eigenfunctions using the recursive condition (C.15). For any time t, time interval $\Delta > 0$, and integer $n \leq M < \eta/\phi^2$, these coefficients correspond to the n-th conditional moment of the transformed process $\hat{B}_{is}(t)$ and satisfy the recursive moment condition

$$\mathbb{E}\left[\hat{B}_{is}(t+\Delta)^n\,\middle|\hat{B}_{is}(t)=b\right] = \exp\{-a_n\Delta\}\sum_{m=0}^{n}\pi_{n,m}b^m - \sum_{m=0}^{n-1}\pi_{n,m}\mathbb{E}\left[\hat{B}_{is}(t+\Delta)^m\,\middle|\hat{B}_{is}(t)=b\right],$$

where the coefficients a_n and $\pi_{n,m}$ $(n,m = 1,\ldots,M)$ are defined above. This equation restates (C.13) in Proposition 2 and is n-th conditional moment recursion referenced in Subsection 4.3.

In practice, it is useful to work with a matrix characterization of these moment conditions by stacking the first N moments in a vector $\mathbf{Y}_{is}(t)$:

$$\mathbf{\Pi}\cdot\mathbb{E}\left[\mathbf{Y}_{is}(t+\Delta)\,\middle|\hat{B}_{is}(t)\right] = \mathbf{\Lambda}(\Delta)\cdot\mathbf{\Pi}\cdot\mathbf{Y}_{is}(t) \tag{C.16}$$

with $\mathbf{Y}_{is}(t) \equiv (1,\hat{B}_{is}(t),\ldots,\hat{B}_{is}(t)^M)'$ and the matrices $\mathbf{\Lambda}(\Delta) = \mathbf{diag}(e^{-a_1\Delta}, e^{-a_2\Delta},\ldots,e^{-a_M\Delta})$ and $\mathbf{\Pi} = (\boldsymbol{\pi}_1,\boldsymbol{\pi}_2,\ldots,\boldsymbol{\pi}_M)'$, where $\boldsymbol{\pi}_m \equiv (\pi_{m,0},\ldots,\pi_{m,m},0,\ldots,0)'$ for each $m = 1,\ldots,M$. In our implementation of the GMM criterion function based on forecast errors, we work with the forecast errors of the linear combination $\mathbf{\Pi}\cdot\mathbf{Y}_{is}(t)$ instead of the forecast errors for $\mathbf{Y}_{is}(t)$. Either estimator is numerically equivalent since the matrix $\mathbf{\Pi}$ is triangular by construction and therefore invertible.

C.3 GMM minimization problem

To derive the GMM estimator (stated in Subsection 4.3), let T_{is} denote the number of time series observations available in industry i and country s. Given sample size of $N = \sum_i\sum_s T_{is}$, our GMM estimator solves the minimization problem

$$(\eta^*,\sigma^*,\phi^*) = \arg\min_{(\eta,\sigma,\phi)}\left(\frac{1}{N}\sum_i\sum_s\sum_\tau g_{is\tau}(\eta,\sigma,\phi)\right)'\mathbf{W}\left(\frac{1}{N}\sum_i\sum_s\sum_\tau g_{is\tau}(\eta,\sigma,\phi)\right) \tag{C.17}$$

for a given weighting matrix $\mathbf{W}$. Being overidentified, we adopt a two-step estimator. On the first step we compute an identity weighting matrix, which provides us with a consistent initial estimate. On the second step we update the weighting matrix to an estimate of the optimal weighting matrix by setting the inverse weighting matrix to $\mathbf{W}^{-1} = (1/N)\sum_i\sum_s\sum_\tau g_{is\tau}(\eta,\sigma,\phi)g_{is\tau}(\eta,\sigma,\phi)'$, which is calculated at the parameter value from the first step. Forman and Sørensen (2008) establish asymptotics for a single time series as $T\to\infty$.[48] For

[48]Our estimator would also fit into the standard GMM framework of Hansen (1982), which establishes consistency and asymptotic normality of our second stage estimator as $IS\to\infty$. To account for the two-step nature of our estimator, we use an asymptotic

estimation, we impose the constraints that $\eta > 0$ and $\sigma^2 > 0$ by reparametrizing the model in terms of $\ln \eta > -\infty$ and $2 \ln \sigma > -\infty$. We evaluate the objective function (C.17) at values of (η, σ, ϕ) by detrending the data at each iteration to obtain $\hat{A}_{is}^{\text{GMM}}(t)$ from equation (19), transforming these variables into their mirror variables $\hat{B}_{is}^{\text{GMM}}(t) = [\hat{A}_{is}^{\text{GMM}}(t)^{-\phi} - 1]/\phi$, and using equation (C.13) to compute forecast errors. Then we calculate the GMM criterion function for each industry and country pair by multiplying these forecast errors by instruments constructed from $\hat{B}_{is}^{\text{GMM}}(t)$, and finally sum over industries and countries to arrive at the value of the GMM objective.

D Correction for Generated Variables in GMM Estimation

D.1 Sampling variation in estimated absolute and comparative advantage

Let $\mathbf{k}_{i \cdot t}$ denote the vector of export capabilities of industry i at time t across countries and $\mathbf{m}_{i \cdot t}$ the vector of importer fixed effects. Denote the set of exporters in the industry in that year with $\mathcal{S}_{it}$ and the set of destinations, to which a country-industry is ships in that year, with $\mathcal{D}_{ist}$. The set of industries active as exporters from source country s in a given year is denoted with $\mathcal{I}_{st}$. Consider the gravity regression (6)

$$\ln X_{isdt} = k_{ist} + m_{idt} + \mathbf{r}'_{sdt}\mathbf{b}_{it} + v_{isdt}.$$

Stacking observations, the regression can be expressed more compactly in matrix notation as

$$\mathbf{x}_{i \cdot \cdot t} = \mathbf{J}_{it}^S \mathbf{k}_{i \cdot t} + \mathbf{J}_{it}^D \mathbf{m}_{i \cdot t} + \mathbf{R}_{\cdot \cdot t}\mathbf{b}_{it} + \mathbf{v}_{i \cdot \cdot t},$$

where $\mathbf{x}_{i \cdot \cdot t}$ is the stacked vector of *log* bilateral exports, $\mathbf{J}_{it}^S$ and $\mathbf{J}_{it}^D$ are matrices of indicators reporting the exporter and importer country by observation, $\mathbf{R}_{\cdot \cdot t}$ is the matrix of bilateral trade cost regressors and $\mathbf{v}_{i \cdot \cdot t}$ is the stacked vector of residuals.

We assume that the two-way least squares dummy variable estimator for each industry time pair it is consistent and asymptotically normal for an individual industry i shipping from source country s to destination d at time t,[49] and state this assumption formally.

Assumption 1. *If $\mathbf{k}_{i \cdot t}^{\text{OLS}}$ is the OLS estimate of $\mathbf{k}_{i \cdot t}$, then*

$$\sqrt{\bar{D}_{it}}(\mathbf{k}_{i \cdot t}^{\text{OLS}} - \mathbf{k}_{i \cdot t}) \xrightarrow{d} \mathcal{N}(\mathbf{0}, \mathbf{\Sigma}_{it}) \text{ as } \bar{D}_{it} \to \infty,$$

where $\bar{D}_{it} \equiv (1/|\mathcal{S}_{it}|) \sum_{s \in \mathcal{S}_{it}} |\mathcal{D}_{ist}|$ is the source-country-average number of countries importing industry i goods in year t and

$$\mathbf{\Sigma}_{it} = \sigma_{it}^2 \left[\lim_{\bar{D}_{it} \to \infty} \frac{1}{\bar{D}_{it}} \left(\mathbf{J}_{it}^S\right)' \mathbf{M}_{it} \left(\mathbf{J}_{it}^S\right) \right]^{-1}$$

with $\sigma_{it}^2 \equiv \mathbb{E}_{it} v_{isdt}^2$,

$$\mathbf{M}_{it} \equiv \mathbf{I}_{|\mathcal{S}_{it}|\bar{D}_{it}} - [\mathbf{J}_{it}^D, \mathbf{R}_{\cdot \cdot t}]\{[\mathbf{J}_{it}^D, \mathbf{R}_{\cdot \cdot t}]'[\mathbf{J}_{it}^D, \mathbf{R}_{\cdot \cdot t}]\}^{-1}[\mathbf{J}_{it}^D, \mathbf{R}_{\cdot \cdot t}]',$$

and $\mathbf{I}_{|\mathcal{S}_{it}|\bar{D}_{it}}$ the identity matrix.

In finite samples, uncertainty as captured by $\mathbf{\Sigma}_{it}$ can introduce sampling variation in second-stage estimation

approximation where each dimension of our panel data gets large simultaneously (see Appendix D).

[49]This high-level assumption can be justified by standard missing-at-random assumptions on the gravity model.

because $\mathbf{k}_{i\cdot t}^{\text{OLS}}$ is a generated variable. To perform an according finite sample correction, we use

$$\mathbf{\Sigma}_{it}^{\text{OLS}} = (\sigma_{it}^{\text{OLS}})^2 \left[\frac{1}{\bar{D}_{it}} \left(\mathbf{J}_{it}^{S} \right)' \mathbf{M}_{it} \left(\mathbf{J}_{it}^{S} \right) \right]^{-1}$$

with $(\sigma_{it}^{\text{OLS}})^2 = (1/|\mathcal{S}_{it}|\bar{D}_{it})(\mathbf{v}_{i\cdot\cdot t}^{\text{OLS}})'\mathbf{v}_{i\cdot\cdot t}^{\text{OLS}}$ to consistently estimate the matrix $\mathbf{\Sigma}_{it}$.

Our second stage estimation uses demeaned first-stage estimates of export capability. For the remainder of this Appendix, we define *log* absolute advantage and *log* comparative advantage in the population as

$$a_{ist} \equiv \ln A_{ist} = k_{ist} - \frac{1}{|\mathcal{S}_{it}|} \sum_{\varsigma \in \mathcal{S}_{it}} k_{i\varsigma t} \quad \text{and} \quad \hat{a}_{ist} \equiv \ln \hat{A}_{ist} = a_{ist} - \frac{1}{|\mathcal{I}_{st}|} \sum_{j \in \mathcal{I}_{st}} a_{jst}. \tag{D.18}$$

Correspondingly, we denote their estimates with a_{ist}^{OLS} and $\hat{a}_{ist}^{\text{OLS}}$.

For each year, let $\mathbf{K}_t^{\text{OLS}}$ denote an $I \times S$ matrix with entries equal to estimated export capability whenever available and equal to zero otherwise, let $\mathbf{H}_t$ record the pattern of non-missing observations and $\mathbf{K}_t$ collect the population values of export capability:

$$[\mathbf{K}_t^{\text{OLS}}]_{is} = \begin{cases} k_{ist}^{\text{OLS}} & s \in \mathcal{S}_{it} \\ 0 & s \notin \mathcal{S}_{it} \end{cases}, \quad [\mathbf{H}_t]_{is} = \begin{cases} 1 & s \in \mathcal{S}_{it} \\ 0 & s \notin \mathcal{S}_{it} \end{cases}, \quad [\mathbf{K}_t]_{is} = \begin{cases} k_{ist} & s \in \mathcal{S}_{it} \\ 0 & s \notin \mathcal{S}_{it}, \end{cases}.$$

where $[\cdot]_{is}$ denotes the specific entry is. Similarly, collect estimates of log absolute advantage into the matrix $\mathbf{A}_t^{\text{OLS}}$ and estimates of log comparative advantage into the matrix $\hat{\mathbf{A}}_t^{\text{OLS}}$:

$$[\mathbf{A}_t^{\text{OLS}}]_{is} = \begin{cases} \ln A_{ist}^{\text{OLS}} & s \in \mathcal{S}_{it} \\ 0 & s \notin \mathcal{S}_{it} \end{cases}, \quad [\hat{\mathbf{A}}_t^{\text{OLS}}]_{is} = \begin{cases} \ln \hat{A}_{ist}^{\text{OLS}} & s \in \mathcal{S}_{it} \\ 0 & s \notin \mathcal{S}_{it} \end{cases}.$$

We maintain the OLS superscripts to clarify that absolute advantage A_{ist}^{OLS} and comparative advantage $\hat{A}_{ist}^{\text{OLS}}$ are generated variables.

The two matrices $\mathbf{A}_t^{\text{OLS}}$ and $\hat{\mathbf{A}}_t^{\text{OLS}}$ are linearly related to the matrix containing our estimates of export capability $\mathbf{K}_t^{\text{OLS}}$. From equation (D.18), the matrix $\mathbf{A}_t^{\text{OLS}}$ is related to $\mathbf{K}_t^{\text{OLS}}$ and $\mathbf{H}_t$ by

$$\mathbf{vec}(\mathbf{A}_t^{\text{OLS}}) = \mathbf{Trans}(I, S) \underbrace{\begin{pmatrix} \mathbf{I}_S - \frac{[\mathbf{H}_t]'_{1\cdot}[\mathbf{H}_t]_{1\cdot}}{[\mathbf{H}_t]_{1\cdot}[\mathbf{H}_t]'_{1\cdot}} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{I}_S - \frac{[\mathbf{H}_t]'_{I\cdot}[\mathbf{H}_t]_{I\cdot}}{[\mathbf{H}_t]_{I\cdot}[\mathbf{H}_t]'_{I\cdot}} \end{pmatrix}}_{\equiv \mathbf{Z}_{IS}(\mathbf{H}_t)} \mathbf{vec}[(\mathbf{K}_t^{\text{OLS}})']. \tag{D.19}$$

Here $\mathbf{vec}(\cdot)$ stacks the columns of a matrix into a vector and $\mathbf{Trans}(I, S)$ is a vectorized-transpose permutation matrix.[50] The function $\mathbf{Z}_{IS}(\mathbf{H}_t)$ maps the matrix $\mathbf{H}_t$ into a block diagonal $IS \times IS$ matrix, which removes the

[50]The vectorized-transpose permutation matrix of type (m, n) is uniquely defined by the relation

$$\mathbf{vec}(\mathbf{B}) = \mathbf{Trans}(m, n)\mathbf{vec}(\mathbf{B}') \quad \forall \mathbf{B} \in \mathbb{R}^{m \times n}.$$

The (ij)-th entry of this matrix is equal to 1 if $j = 1 + m(i-1) - (mn-1)\text{floor}((i-1)/n)$ and 0 otherwise.

global industry average across countries. The matrix of comparative advantage estimates is then:

$$\mathbf{vec}(\hat{\mathbf{A}}_t^{\text{OLS}}) = \underbrace{\begin{pmatrix} \mathbf{I}_I - \frac{[\mathbf{H}_t']_{1\cdot}'[\mathbf{H}_t']_{1\cdot}}{[\mathbf{H}_t']_{1\cdot}[\mathbf{H}_t']_{1\cdot}'} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{I}_I - \frac{[\mathbf{H}_t']_{S\cdot}'[\mathbf{H}_t']_{S\cdot}}{[\mathbf{H}_t']_{S\cdot}[\mathbf{H}_t']_{S\cdot}'} \end{pmatrix}}_{\equiv \mathbf{Z}_{SI}(\mathbf{H}_t')} \mathbf{vec}(\mathbf{A}_t^{\text{OLS}}) \equiv \mathbf{Z}_{SI}(\mathbf{H}_t')\mathbf{vec}(\mathbf{A}_t^{\text{OLS}}). \qquad \text{(D.20)}$$

The function $\mathbf{Z}_{SI}(\mathbf{H}_t')$ maps the matrix $\mathbf{H}_t$ into a block diagonal $SI \times SI$ matrix, which removes the national average across industries.

For simplicity, we assume that the sampling variation in export capability estimates is uncorrelated across industries and years.

Assumption 2. *For any $(it) \neq (jT)$, $\mathbb{E}(\mathbf{k}_{i\cdot t}^{\text{OLS}} - \mathbf{k}_{i\cdot t})(\mathbf{k}_{j\cdot T}^{\text{OLS}} - \mathbf{k}_{j\cdot T})' = \mathbf{0}$.*

We then have the following result.

Lemma 3. *Suppose Assumptions 1 and 2 hold and that there is an $\omega_{it} > 0$ for each (it) so that $\lim_{D\to\infty} \bar{D}_{it}/D = \omega_{it}$. Then*

$$\sqrt{D}[\mathbf{vec}(\mathbf{A}_t^{\text{OLS}}) - \mathbf{Trans}(I,S)\mathbf{Z}_{IS}(\mathbf{H}_t)\mathbf{vec}[(\mathbf{K}_t^{\text{OLS}})']] \xrightarrow{d} \mathcal{N}(\mathbf{0}, \mathbf{Trans}(I,S)\mathbf{Z}_{IS}(\mathbf{H}_t)\,\mathbf{\Sigma}_t^*\,\mathbf{Z}_{IS}(\mathbf{H}_t)'\mathbf{Trans}(I,S)')$$

and

$$\begin{aligned} &\sqrt{D}\{\mathbf{vec}(\hat{\mathbf{A}}_t^{\text{OLS}}) - \mathbf{Z}_{SI}(\mathbf{H}_t')\mathbf{Trans}(I,S)\mathbf{Z}_{IS}(\mathbf{H}_t)\mathbf{vec}[(\mathbf{K}_t^{\text{OLS}})'] \\ &\qquad \xrightarrow{d} \mathcal{N}(\mathbf{0}, \mathbf{Z}_{SI}(\mathbf{H}_t')\mathbf{Trans}(I,S)\mathbf{Z}_{IS}(\mathbf{H}_t)\,\mathbf{\Sigma}_t^*\,\mathbf{Z}_{IS}(\mathbf{H}_t)'\mathbf{Trans}(I,S)'\mathbf{Z}_{SI}(\mathbf{H}_t')') \end{aligned}$$

with

$$\mathbf{\Sigma}_t^* \equiv \begin{pmatrix} \omega_{1t}^{-1}\mathbf{\Sigma}_{1t}^* & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \omega_{It}^{-1}\mathbf{\Sigma}_{It}^* \end{pmatrix}$$

where the s-th column of $\mathbf{\Sigma}_{it}^*$ is equal to country s's corresponding column in $\mathbf{\Sigma}_{it}$ whenever export capability is estimated for (ist) and is a vector of zeros otherwise.

Proof. Assumptions 1 and 2 along with $\bar{D}_{it} \to D \to \infty$ for all (it) implies that $\sqrt{D}(\mathbf{vec}[(\mathbf{K}_t^{\text{OLS}})'] - \mathbf{vec}[\mathbf{K}_t']) \xrightarrow{d} \mathcal{N}(\mathbf{0}, \mathbf{\Sigma}_t^*)$. The results then follow from equation (D.19) and equation (D.20). □

D.2 Second-stage generated variable correction

We estimate two time series models which both can be implemented as GMM estimators. For brevity, we focus on GLD estimation here. (We present the case of OLS estimation of the decay regression in the Supplementary Material (Section S.2), which simply uses a different GMM criterion and absolute advantage as data instead of comparative advantage.) GLD estimation is based on a conditional moment of the form:

$$\mathbf{0} = \mathbb{E}_{is,t-\Delta}\,\mathbf{g}\left(\boldsymbol{\theta}, \hat{a}_{ist}, \hat{a}_{is,t-\Delta}\right), \qquad \text{(D.21)}$$

where $\boldsymbol{\theta} = (\eta, \sigma, \phi)'$ is the vector of parameters. In our overidentified GMM estimator, $\mathbf{g}$ is a column vector of known continuously differentiable functions (moment conditions) for any time lag $\Delta > 0$.

The moment conditions apply to any instant in continuous time, but our data come in discrete annual observations for a finite period of years. To account for missing data, let $\mathcal{S}_{it}^P \subset \mathcal{S}_{it}$ denote the set of countries that were *previously* observed to export good i and that are still exporting good i at current time t: $\mathcal{S}_{it}^P \equiv \{s \in \mathcal{S}_{it} \mid \exists\, \tau^P < t \,\text{s.t.}\, s \in \mathcal{S}_{i\tau^P}\}$. Similarly, let $\mathcal{S}_{it}^F \equiv \{s \in \mathcal{S}_{it} \mid \exists\, \tau^F > t \,\text{s.t.}\, s \in \mathcal{S}_{i\tau^F}\}$ be current exporter countries that ship good i to at least one destination also some *future* year. Denote the most recent prior period in which s exported in industry i by $\tau_{ist}^P \equiv \sup\{\tau^P < t \mid s \in \mathcal{S}_{i\tau^P}\}$ and the most recent future period in which s will export by $\tau_{ist}^F \equiv \inf\{\tau^F > t \mid s \in \mathcal{S}_{i\tau^F}\}$. We will use these objects to keep track of timing.

For instance, for each $i = 1, \dots, I$, $t = 2, \dots, T$, and $s \in \mathcal{S}_{it}^P$ we can design a GMM criterion based on the following conditional moment:

$$\mathbb{E}_{i,s,\tau_{ist}^P} \mathbf{g}\left(\boldsymbol{\theta}, \hat{a}_{ist}, \hat{a}_{is\tau_{ist}^P}\right) = \mathbf{0}.$$

Our finite sample analog for second-stage estimation is:

$$\frac{1}{I(T-1)} \sum_{i=1}^{I} \sum_{t=2}^{T} \frac{1}{|\mathcal{S}_{it}^P|} \sum_{s \in \mathcal{S}_{it}^P} \mathbf{g}_{ist}(\boldsymbol{\theta}) \quad \text{with } \mathbf{g}_{ist}(\boldsymbol{\theta}) \equiv \mathbf{g}\left(\boldsymbol{\theta}, \hat{a}_{ist}^{\text{OLS}}, \hat{a}_{is\tau_{ist}^P}^{\text{OLS}}\right),$$

where $|\mathcal{S}_{it}^P|$ is the number of exporters in industry i at time t that were also observed exporting good i at a previous time.

The effective sample size for the second stage is $N \equiv \sum_{t=1}^{I} \sum_{t=1}^{T-1} |\mathcal{S}_{it}|$ and the GMM criterion can be expressed as

$$Q_N(\boldsymbol{\theta}; \mathbf{W}) = \left(\frac{1}{N} \sum_{i=1}^{I} \sum_{t=2}^{T} \sum_{s \in \mathcal{S}_{it}^P} \frac{N}{I|\mathcal{S}_{it}^P|(T-1)} \mathbf{g}_{ist}(\boldsymbol{\theta})\right)' \mathbf{W} \left(\frac{1}{N} \sum_{i=1}^{I} \sum_{t=2}^{T} \sum_{s \in \mathcal{S}_{it}^P} \frac{N}{I|\mathcal{S}_{it}^P|(T-1)} \mathbf{g}_{ist}(\boldsymbol{\theta})\right)$$

where $\mathbf{W}$ is a weighting matrix.

In order to get consistency, we assume that all dimensions of our data are large as N gets large.

Assumption 3. *As $N \to \infty$ we have*

1. $D \to \infty$;
2. $\forall (it)\ \exists \omega_{it} > 0$ *so that* $\bar{D}_{it}/D \to \omega_{it}$, $N/[I|\mathcal{S}_{it}^P|(T-1)] \to 1$, *and* $|\mathcal{S}_{it}| \to \infty$;
3. $\forall (st)\ |\mathcal{I}_{st}| \to \infty$;
4. $T \to \infty$.

Letting $D \to \infty$ and $\bar{D}_{it}/D \to \omega_{it} > 0$ ensures that we consistently estimate $\mathbf{k}_{i \cdot t}$ on the first stage and we can use Lemma 3 for the first stage sampling distribution of comparative advantage. Then, letting $|\mathcal{S}_{it}| \to \infty$ ensures that we consistently estimate absolute advantage and $|\mathcal{I}_{st}| \to \infty$ lets us consistently estimate comparative advantage. The asymptotic results of Forman and Sørensen (2008) apply under the assumption that $T \to \infty$.

Under the maintained assumptions, we get the following consistency result.

Proposition 3. *Suppose that*

1. $\boldsymbol{\theta} \in \Theta$ *for some compact set* Θ;
2. *for any* $\Delta > 0$, *there is a unique* $\boldsymbol{\theta}_0 \in \Theta$ *such that*

$$\mathbf{0} = \mathbb{E}\mathbf{g}\left(\boldsymbol{\theta}_0, \hat{a}_{ist}, \hat{a}_{is,t-\Delta}\right);$$

3. *for any given positive definite matrix W and for each N, there is a unique minimizer of $Q_N(\theta; \mathbf{W})$ given by $\hat{\theta}_N$;*

4. *both $\mathbb{E}_{it} k_{ist}$ and $\mathbb{E}_{st} k_{ist}$ exist and are finite.*

Then, under Assumptions 1 and 3, we have $\hat{\theta}_N \xrightarrow{p} \theta_0$.

Proof. The proof follows from a standard consistency argument for extremum estimators (see e.g. Newey and McFadden 1994). Given (a) compactness of the parameter space, (b) the continuity of the GMM objective, and (c) the existence of moments as in Forman and Sørensen (2008), we get a uniform law of large numbers for the objective function on the parameter space as $N \to \infty$. The GLD estimator is then consistent under the assumption that the model is identified, provided that we consistently estimate comparative advantage. The consistency of our comparative advantage estimates follows from the strong law of large numbers given Assumption 3 and the existence and finiteness of $\mathbb{E}_{it} k_{ist}$ and $\mathbb{E}_{st} k_{ist}$. □

Proposition 4. *Under the conditions of Proposition 3 and Assumptions 1, 2, and 3 we have*

$$\sqrt{N}(\hat{\theta}_N - \theta_0) \xrightarrow{d} \mathcal{N}(\mathbf{0}, (\mathbf{\Lambda}'\mathbf{W}\mathbf{\Lambda})^{-1}\mathbf{\Lambda}'\mathbf{W}(\mathbf{\Xi} + \mathbf{\Omega})\,\mathbf{W}\,\mathbf{\Lambda}(\mathbf{\Lambda}'\mathbf{W}\mathbf{\Lambda})^{-1}),$$

where

$$\begin{aligned}
\mathbf{\Lambda} &= \mathbb{E}\frac{\partial}{\partial\theta}\mathbf{g}\left(\theta_0, \hat{a}_{ist}, \hat{a}_{is\tau^P_{ist}}\right), \\
\mathbf{\Xi} &= \mathbb{E}\mathbf{g}\left(\theta_0, \hat{a}_{ist}, \hat{a}_{is\tau^P_{ist}}\right)\mathbf{g}\left(\theta_0, \hat{a}_{ist}, \hat{a}_{is\tau^P_{ist}}\right)', \\
\mathbf{\Omega} &= \lim_{N\to\infty}\frac{1}{ND}\sum_{t=1}^{T}\mathbf{G}_t\mathbf{Z}_{SI}(\mathbf{H}'_t)\mathbf{Trans}(I,S)\mathbf{Z}_{IS}(\mathbf{H}_t)\,\mathbf{\Sigma}^*_t\,\mathbf{Z}_{IS}(\mathbf{H}_t)'\mathbf{Trans}(I,S)'\mathbf{Z}_{SI}(\mathbf{H}'_t)'\mathbf{G}'_t
\end{aligned}$$

for a $\mathbf{G}_t$ matrix of weighted Jacobians of $\mathbf{g}_{ist}(\theta)$, as defined below.

Proof. To get a correction for first stage sampling variation, we use a mean-value expansion of the GMM criterion. Given continuous differentiability of the moment function $\mathbf{g}_{ist}(\theta)$ and the fact that $\hat{\theta}_N$ maximizes $Q_N(\theta; \mathbf{W})$ we must have

$$\begin{aligned}
\mathbf{0} &= \frac{\partial}{\partial\theta}Q_N(\hat{\theta}_N; \mathbf{W}) \\
&= \left(\frac{1}{N}\sum_{i=1}^{I}\sum_{t=2}^{T}\sum_{s\in\mathcal{S}^P_{it}}\frac{N}{I|\mathcal{S}^P_{it}|(T-1)}\frac{\partial}{\partial\theta}\mathbf{g}_{ist}(\hat{\theta}_N)\right)'\mathbf{W}\left(\frac{1}{N}\sum_{i=1}^{I}\sum_{t=2}^{T}\sum_{s\in\mathcal{S}^P_{it}}\frac{N}{I|\mathcal{S}^P_{it}|(T-1)}\mathbf{g}_{ist}(\hat{\theta}_N)\right).
\end{aligned}$$

The criterion function $\mathbf{g}$ is continuously differentiable. Therefore, by the mean value theorem, there exist random variables $\tilde{\theta}_N$ and $\tilde{a}_{ist}$ such that $|\tilde{\theta}_N - \theta_0| \le |\hat{\theta}_N - \theta_0|$, $|\tilde{a}_{ist} - \hat{a}_{ist}| \le |\hat{a}^{\text{OLS}}_{ist} - \hat{a}_{ist}|$, and

$$\begin{aligned}
\mathbf{g}(\hat{\theta}_N; ist) = &\underbrace{\mathbf{g}\left(\theta_0, \hat{a}_{ist}, \hat{a}_{is\tau^P_{ist}}\right)}_{\equiv\mathbf{G}^0_{ist}} + \underbrace{\frac{\partial}{\partial\theta}\mathbf{g}\left(\theta, \tilde{a}_{ist}, \tilde{a}_{is\tau^P_{ist}}\right)\Big|_{\theta=\tilde{\theta}_N}}_{\equiv\tilde{\mathbf{G}}^1_{ist}}(\hat{\theta}_N - \theta_0) \\
&+ \underbrace{\frac{\partial}{\partial a}\mathbf{g}\left(\tilde{\theta}_N, a, \tilde{a}_{is\tau^P_{ist}}\right)\Big|_{a=\tilde{a}_{ist}}}_{\equiv\tilde{\mathbf{G}}^2_{ist}}(\hat{a}^{\text{OLS}}_{ist} - \hat{a}_{ist}) + \underbrace{\frac{\partial}{\partial a^P}\mathbf{g}\left(\tilde{\theta}_N, \tilde{a}_{ist}, a^P\right)\Big|_{a^P=\tilde{a}_{is\tau^P_{ist}}}}_{\equiv\tilde{\mathbf{G}}^3_{ist}}\left(\hat{a}^{\text{OLS}}_{is\tau^P_{ist}} - \hat{a}_{is\tau^P_{ist}}\right).
\end{aligned}$$

Then,

$$\mathbf{0} = \tilde{\mathbf{\Lambda}}_N' \mathbf{W} \frac{1}{N} \sum_{i=1}^{I} \sum_{t=2}^{T} \sum_{s \in \mathcal{S}_{it}^P} \frac{N}{I|\mathcal{S}_{it}^P|(T-1)} \left[\mathbf{G}_{ist}^0 + \tilde{\mathbf{G}}_{ist}^1 (\hat{\boldsymbol{\theta}}_N - \boldsymbol{\theta}_0) + \tilde{\mathbf{G}}_{ist}^2 (\hat{a}_{ist}^{\text{OLS}} - \hat{a}_{ist}) + \tilde{\mathbf{G}}_{ist}^3 (\hat{a}_{is\tau_{ist}^P}^{\text{OLS}} - \hat{a}_{is\tau_{ist}^P}) \right]$$

where $\tilde{\mathbf{\Lambda}}_N = \frac{1}{I(T-1)} \sum_{i=1}^{I} \sum_{t=2}^{T} \frac{1}{|\mathcal{S}_{it}^P|} \sum_{s \in \mathcal{S}_{it}^P} \tilde{\mathbf{G}}_{ist}^1$.

Solving for $\hat{\boldsymbol{\theta}}_N - \boldsymbol{\theta}_0$ and multiplying by $\sqrt{N}$, we obtain

$$\sqrt{N}(\hat{\boldsymbol{\theta}}_N - \boldsymbol{\theta}_0) =$$

$$- \left[\tilde{\mathbf{\Lambda}}_N' \mathbf{W} \tilde{\mathbf{\Lambda}}_N \right]^{-1} \tilde{\mathbf{\Lambda}}_N' \mathbf{W} \frac{1}{\sqrt{N}} \sum_{i=1}^{I} \sum_{t=2}^{T} \sum_{s \in \mathcal{S}_{it}^P} \frac{N}{I|\mathcal{S}_{it}^P|(T-1)} \left[\mathbf{G}_{ist}^0 + \tilde{\mathbf{G}}_{ist}^2 (\hat{a}_{ist}^{\text{OLS}} - \hat{a}_{ist}) + \tilde{\mathbf{G}}_{ist}^3 (\hat{a}_{is\tau_{ist}^P}^{\text{OLS}} - \hat{a}_{is\tau_{ist}^P}) \right].$$

Note that the set $\mathcal{S}_{i1}^P$ is empty since no country is observed exporting in years before the first sample year and $\mathcal{S}_{iT}^F$ is empty since no country is observed exporting after the final sample year. Moreover,

$$\begin{aligned}
\tilde{\mathbf{\Lambda}}_N &\xrightarrow{p} \Lambda \equiv \mathbb{E} \frac{\partial}{\partial \boldsymbol{\theta}} \mathbf{g} \left(\boldsymbol{\theta}_0, \hat{a}_{ist}, \hat{a}_{is\tau_{ist}^P} \right) \\
\tilde{\mathbf{G}}_{ist}^2 &\xrightarrow{p} \mathbf{G}_{ist}^2 \equiv \left. \frac{\partial}{\partial a} \mathbf{g} \left(\boldsymbol{\theta}_0, a, \hat{a}_{is\tau_{ist}^P} \right) \right|_{a = \hat{a}_{ist}} \\
\tilde{\mathbf{G}}_{ist}^3 &\xrightarrow{p} \mathbf{G}_{ist}^3 \equiv \left. \frac{\partial}{\partial a^P} \mathbf{g} \left(\boldsymbol{\theta}_0, \hat{a}_{ist}, a^P \right) \right|_{a^P = \hat{a}_{is\tau_{ist}^P}}
\end{aligned}$$

because $\hat{\boldsymbol{\theta}}_N$ and $\hat{a}_{ist}^{\text{OLS}}$ are consistent and $\mathbf{g}$ is the continuously differentiable.

As a result, we can re-write the sum as

$$\begin{aligned}
&\frac{1}{\sqrt{N}} \sum_{i=1}^{I} \sum_{t=2}^{T} \sum_{s \in \mathcal{S}_{it}^P} \frac{N}{I|\mathcal{S}_{it}^P|(T-1)} \left[\mathbf{G}_{ist}^0 + \tilde{\mathbf{G}}_{ist}^2 (\hat{a}_{ist}^{\text{OLS}} - \hat{a}_{ist}) + \tilde{\mathbf{G}}_{ist}^3 (\hat{a}_{is\tau_{ist}^P}^{\text{OLS}} - \hat{a}_{is\tau_{ist}^P}) \right] \\
&= \frac{1}{\sqrt{N}} \sum_{i=1}^{I} \sum_{t=2}^{T} \sum_{s \in \mathcal{S}_{it}^P} \frac{N}{I|\mathcal{S}_{it}^P|(T-1)} \left[\mathbf{G}_{ist}^0 + \mathbf{G}_{ist}^2 (\hat{a}_{ist}^{\text{OLS}} - \hat{a}_{ist}) + \mathbf{G}_{ist}^3 (\hat{a}_{is\tau_{ist}^P}^{\text{OLS}} - \hat{a}_{is\tau_{ist}^P}) \right] + o_p(1) \\
&= \frac{1}{\sqrt{N}} \sum_{i=1}^{I} \sum_{t=2}^{T} \sum_{s \in \mathcal{S}_{it}^P} \frac{N}{I|\mathcal{S}_{it}^P|(T-1)} \mathbf{G}_{ist}^0 + o_p(1) \\
&\quad + \frac{1}{\sqrt{N}} \sum_{t=1}^{T} \underbrace{\sum_{i=1}^{I} \sum_{s=1}^{S} \left[\mathbf{1}\{s \in \mathcal{S}_{it}^P\} \frac{N}{I|\mathcal{S}_{it}^P|(T-1)} \mathbf{G}_{ist}^2 + \mathbf{1}\{s \in \mathcal{S}_{i\tau_{ist}^F}^0\} \frac{N}{I|\mathcal{S}_{i\tau_{ist}^F}^0|(T-1)} \mathbf{G}_{is\tau_{ist}^F}^3 \right] (\hat{a}_{ist}^{\text{OLS}} - \hat{a}_{ist})}_{\equiv \mathbf{L}_t},
\end{aligned}$$

using the fact that $\tau^F = \tau_{ist}^F \Leftrightarrow \tau_{is\tau^F}^P = t$.

The term $\mathbf{L}_t$ is a vector and a linear function of the entries of the matrix $\hat{\mathbf{A}}_t^{\text{OLS}} - \hat{\mathbf{A}}_t$. This vector can also be expressed as

$$\mathbf{L}_t = \mathbf{G}_t \mathbf{vec}(\hat{\mathbf{A}}_t^{\text{OLS}} - \hat{\mathbf{A}}_t),$$

and the matrix $\mathbf{G}_t$ has entries

$$[\mathbf{G}_t]_{\cdot j} = \mathbf{1}\left\{s(j) \in \mathcal{S}^P_{i(j),t}\right\} \frac{N}{I\left|\mathcal{S}^P_{i(j),t}\right|(T-1)} \mathbf{G}^2_{i(j),s(j),t} + \mathbf{1}\left\{s(j) \in \mathcal{S}^F_{i(j),\tau^F_{i(j),s(j),t}}\right\} \frac{N}{I\left|\mathcal{S}^F_{i(j),\tau^F_{i(j),s(j),t}}\right|(T-1)} \mathbf{G}^3_{i(j),s(j),\tau^F_{i(j),s(j),t}}$$

for

$$i(j) = 1 + (j \mod S), \quad s(j) = 1 + \text{floor}((j-1)/S).$$

We can now re-write the sum as

$$\frac{1}{\sqrt{N}} \sum_{i=1}^{I} \sum_{t=2}^{T} \sum_{s \in \mathcal{S}^P_{it}} \frac{N}{I|\mathcal{S}^P_{it}|(T-1)} \left[\mathbf{G}^0_{ist} + \tilde{\mathbf{G}}^2_{ist}(\hat{a}^{\text{OLS}}_{ist} - \hat{a}_{ist}) + \tilde{\mathbf{G}}^3_{ist}(\hat{a}^{\text{OLS}}_{is\tau^P_{ist}} - \hat{a}_{is\tau^P_{ist}})\right]$$

$$= \frac{1}{\sqrt{N}} \sum_{i=1}^{I} \sum_{t=2}^{T} \sum_{s \in \mathcal{S}^P_{it}} \frac{N}{I|\mathcal{S}^P_{it}|(T-1)} \mathbf{G}^0_{ist} + \frac{1}{\sqrt{ND}} \sum_{t=1}^{T} \mathbf{G}_t \sqrt{D}\,\mathbf{vec}(\hat{\mathbf{A}}^{\text{OLS}}_t - \hat{\mathbf{A}}_t) + o_p(1).$$

The first term is asymptotically normal under the results of Forman and Sørensen (2008). The second term is asymptotically normal because $\hat{\mathbf{A}}^{\text{OLS}}_t$ is asymptotically normal by Lemma 3. □

For an adaption of the GMM generated-variable correction to second-stage OLS estimation, see the Supplementary Material (Section S.2).

E Additional Evidence

In this Appendix, we report additional evidence to complement the reported findings in the text.

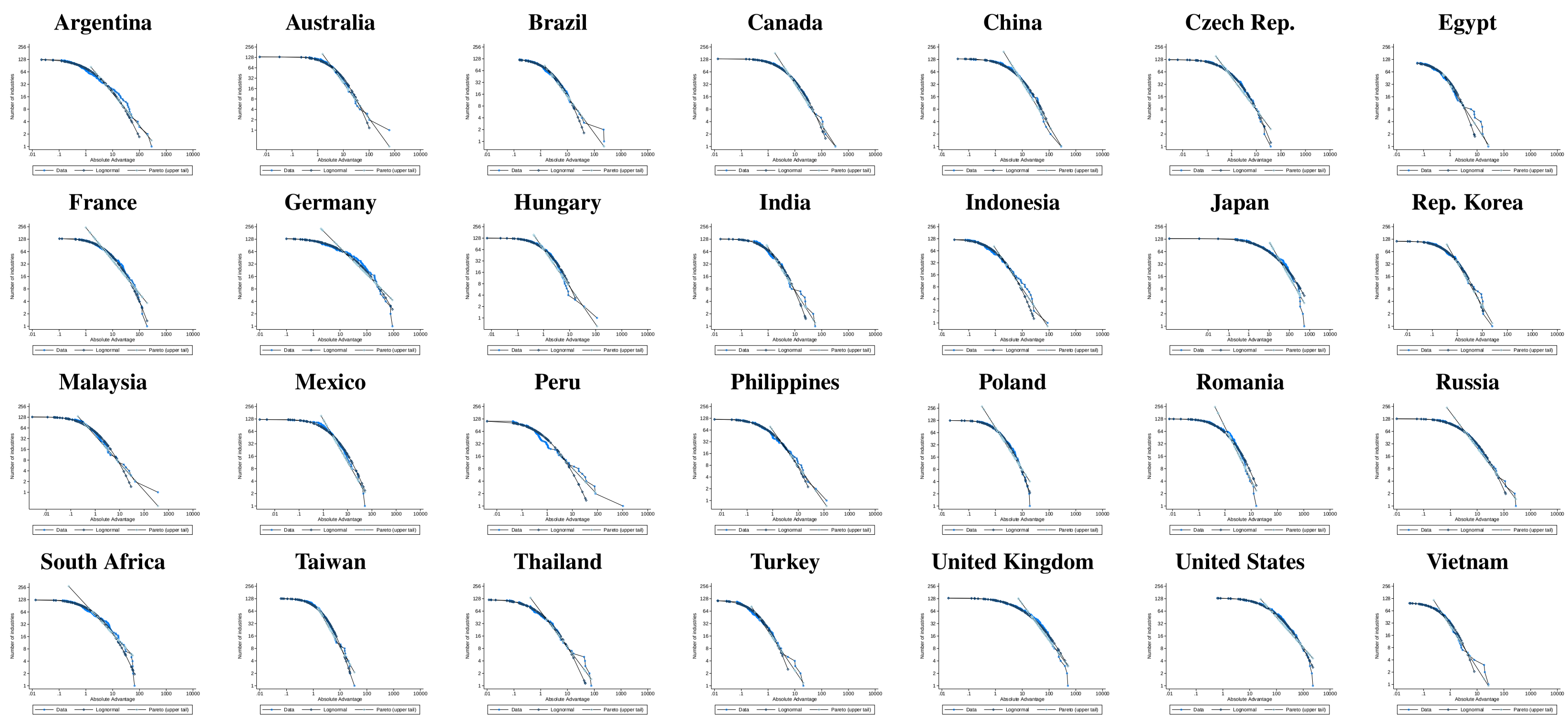
E.1 Cumulative probability distribution of absolute advantage

Figures A1, A2 and **A3** extend **Figure 2** in the text and plot, for 28 countries in 1967, 1987 and 2007, the log number of a source country s's industries that have at least a given level of absolute advantage in year t against that log absolute advantage level $\ln A_{ist}$ for industries i. The figures also graph the fit of absolute advantage in the cross section to a Pareto distribution and to a log normal distribution using maximum likelihood, where each cross sectional distribution is fit separately for each country in each year (such that the number of parameters estimated equals the number of parameters for a distribution × number of countries × number of years). In the Supplementary Material (Section S.6) we show comparable cumulative probability distributions of log absolute advantage for PPML-based exporter capability, the Balassa RCA index, and varying industry aggregates of OLS-based exporter capability.

E.2 GLD predicted cumulative probability distributions of absolute advantage

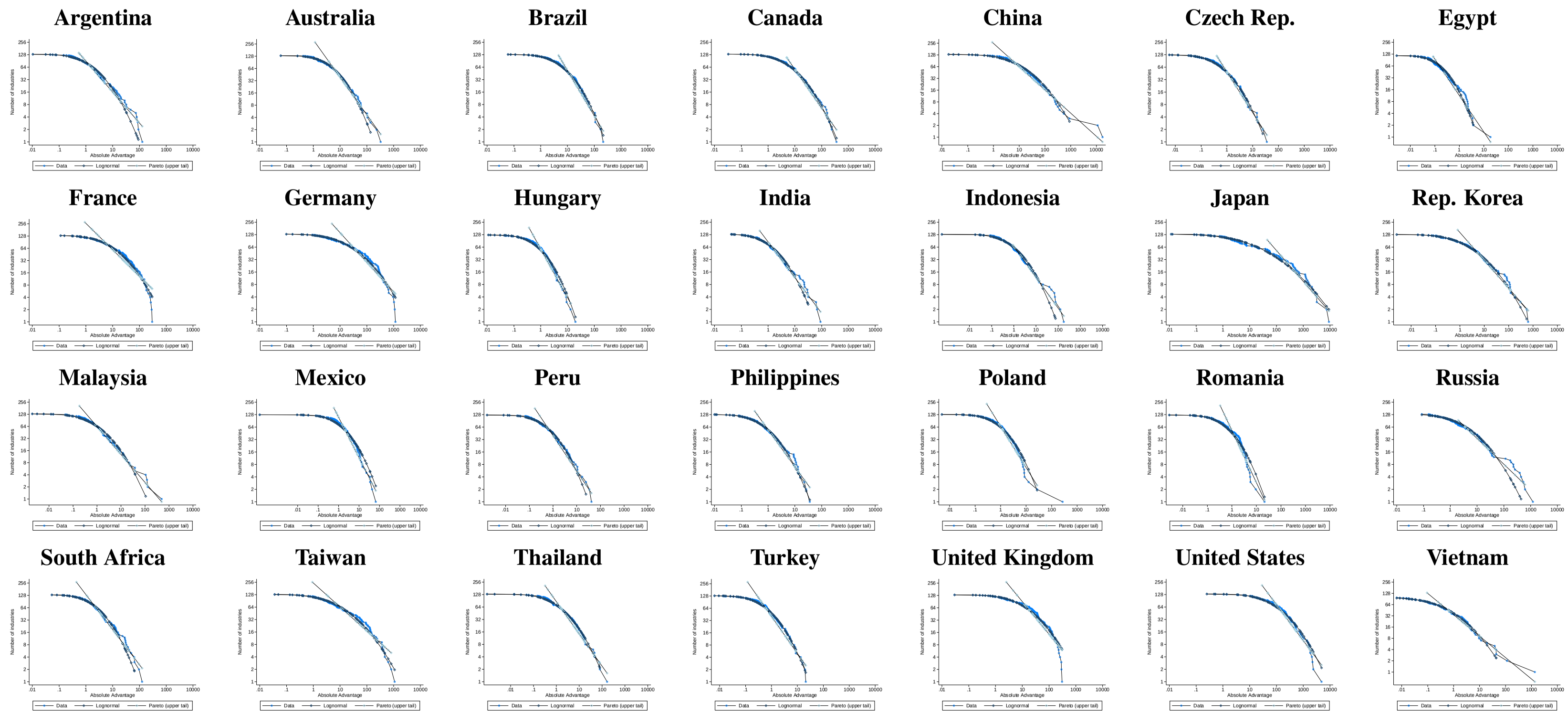
Figures A4, A5 and **A6** present plots for the same 28 countries in 1967, 1987 and 2007 as shown before (in **Figures A1, A2** and **A3**), using log absolute advantage from OLS-based exporter capability. **Figures A4** through **A6** contrast graphs of the actual data with the GLD implied predictions and show a lose fit.

Figure A1: **Cumulative Probability Distribution of Absolute Advantage for 28 Countries in 1967**



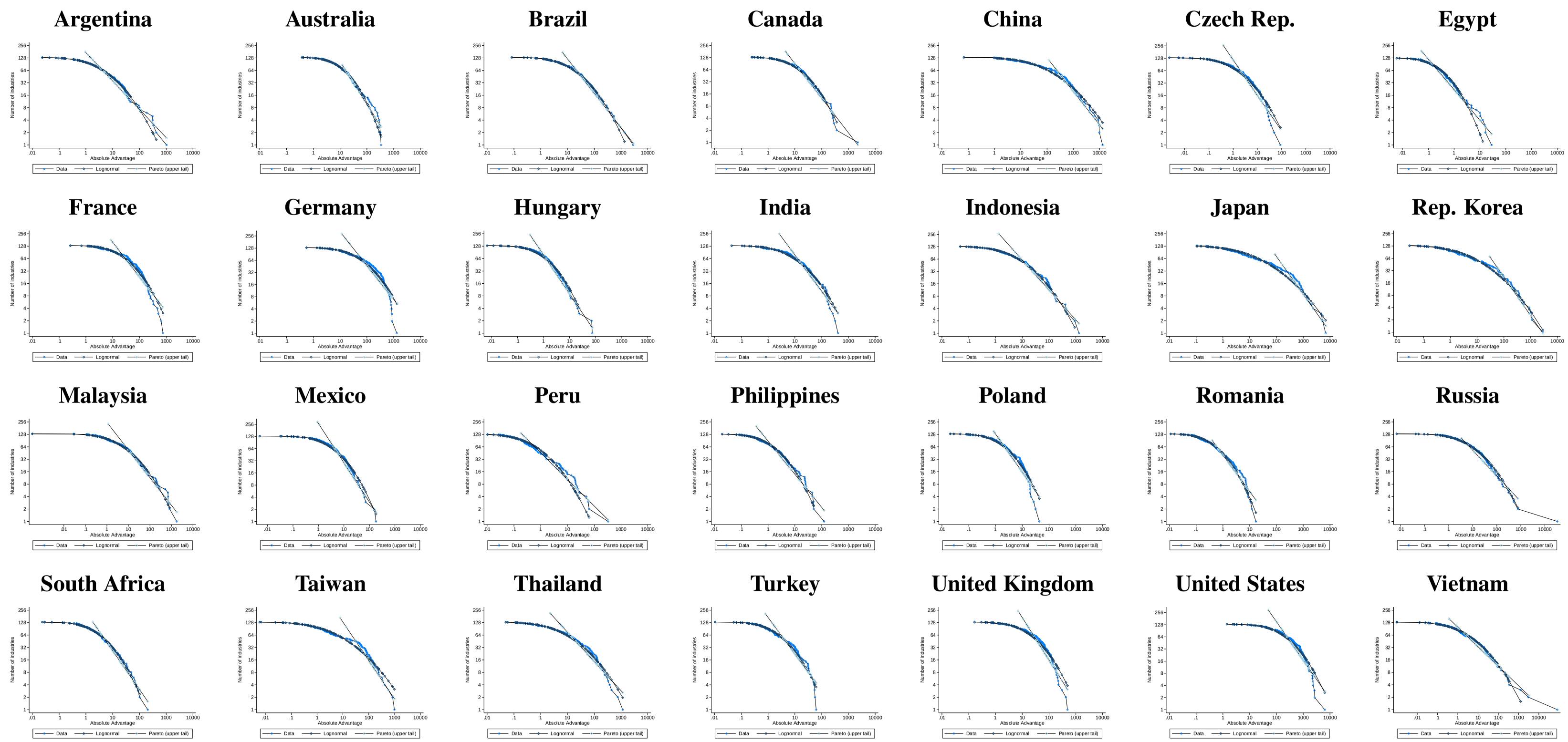
Source: WTF (Feenstra et al. 2005, updated through 2008) for 133 time-consistent industries in 90 countries from 1965-1967 and CEPII.org; three-year means of OLS gravity measures of export capability (log absolute advantage) $k = \ln A$ from (6).
Note: The graphs show the frequency of industries (the cumulative probability $1 - F_A(a)$ times the total number of industries $I = 133$) on the vertical axis plotted against the level of absolute advantage a (such that $A_{ist} \geq a$) on the horizontal axis. Both axes have a log scale. The fitted Pareto and log normal distributions are based on maximum likelihood estimation by country s in year $t = 1967$ (Pareto fit to upper five percentiles only).

Figure A2: **Cumulative Probability Distribution of Absolute Advantage for 28 Countries in 1987**



Source: WTF (Feenstra et al. 2005, updated through 2008) for 133 time-consistent industries in 90 countries from 1985-1987 and CEPII.org; three-year means of OLS gravity measures of export capability (log absolute advantage) $k = \ln A$ from (6).
Note: The graphs show the frequency of industries (the cumulative probability $1 - F_A(a)$ times the total number of industries $I = 133$) on the vertical axis plotted against the level of absolute advantage a (such that $A_{ist} \geq a$) on the horizontal axis. Both axes have a log scale. The fitted Pareto and log normal distributions are based on maximum likelihood estimation by country s in year $t = 1987$ (Pareto fit to upper five percentiles only).

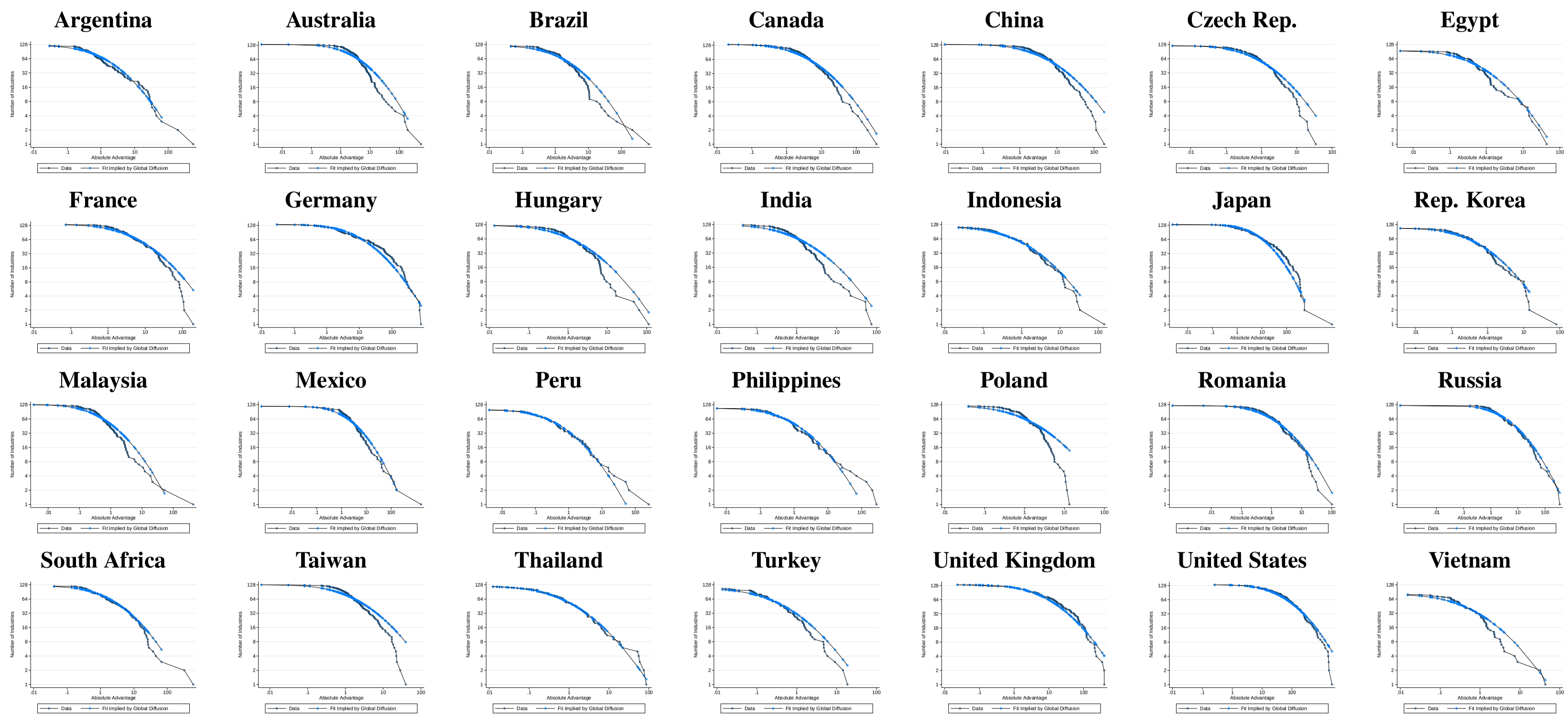
Figure A3: **Cumulative Probability Distribution of Absolute Advantage for 28 Countries in 2007**



Source: WTF (Feenstra et al. 2005, updated through 2008) for 133 time-consistent industries in 90 countries from 2005-2007 and CEPII.org; three-year means of OLS gravity measures of export capability (log absolute advantage) $k = \ln A$ from (6).

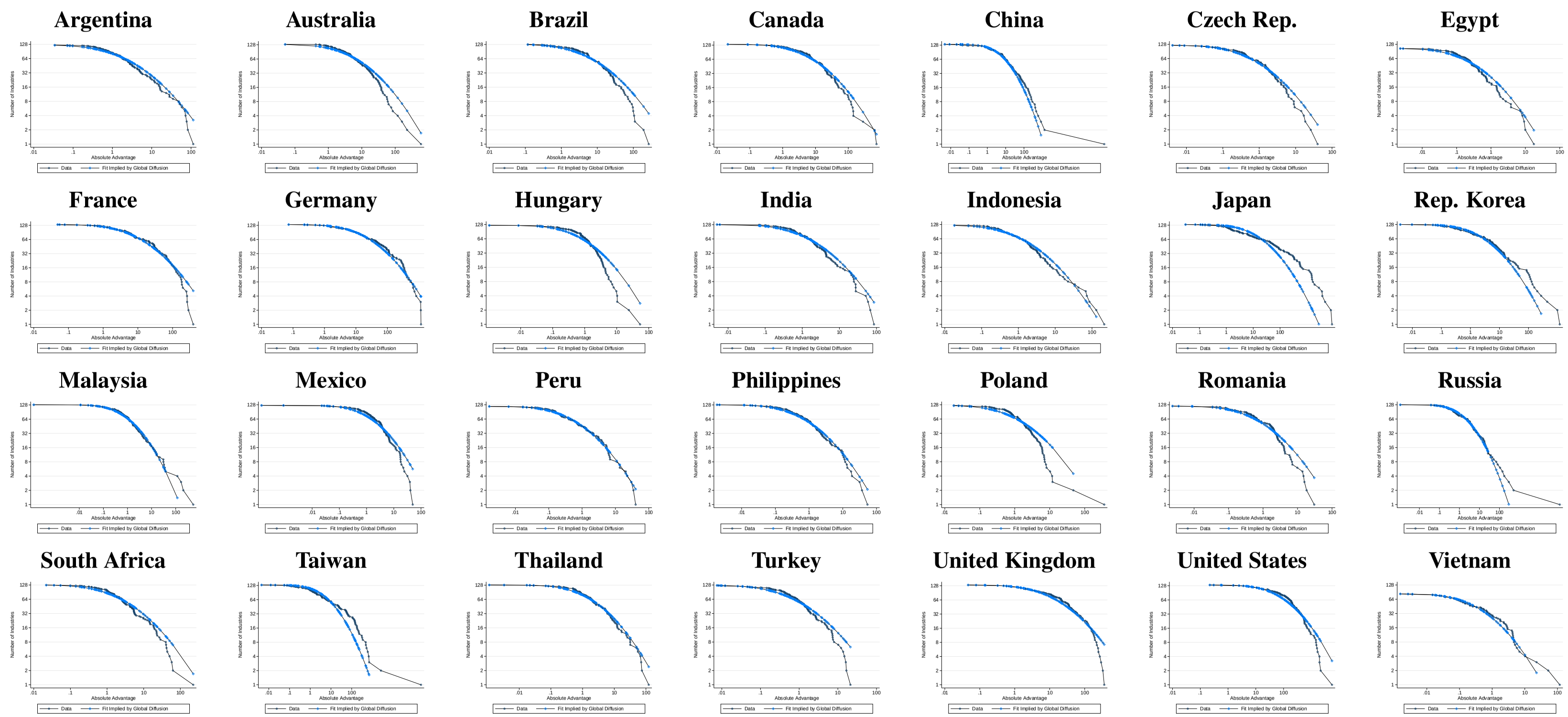
Note: The graphs show the frequency of industries (the cumulative probability $1 - F_A(a)$ times the total number of industries $I = 133$) on the vertical axis plotted against the level of absolute advantage a (such that $A_{ist} \geq a$) on the horizontal axis. Both axes have a log scale. The fitted Pareto and log normal distributions are based on maximum likelihood estimation by country s in year $t = 2007$ (Pareto fit to upper five percentiles only).

Figure A4: **Diffusion Predicted and Observed Cumulative Probability Distributions of Absolute Advantage in 1967**



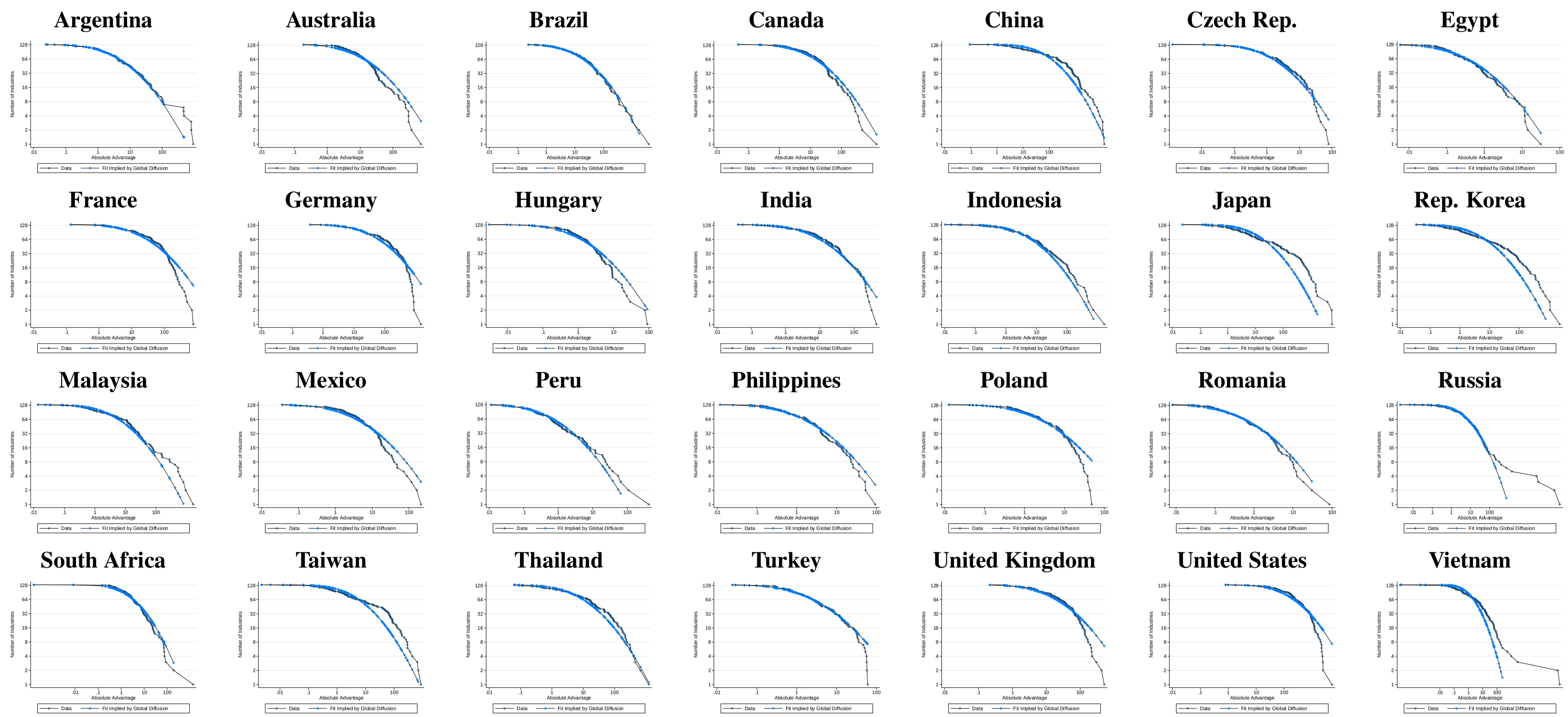
Source: WTF (Feenstra et al. 2005, updated through 2008) for 133 time-consistent industries in 90 countries from 1962-2007 and CEPII.org; OLS gravity measures of export capability (log absolute advantage) $k = \ln A$ from (6).
Note: The graphs show the observed and predicted frequency of industries (the cumulative probability $1 - F_A(a)$ times the total number of industries $I = 133$) on the vertical axis plotted against the level of absolute advantage a (such that $A_{ist} \geq a$) on the horizontal axis. Both axes have a log scale. The predicted frequencies are based on the GMM estimates of the comparative advantage diffusion (17) in Table 2 (parameters η and ϕ in column 1) and the inferred country-specific stochastic trend component $\ln Z_{st}$ from (18), which horizontally shifts the distributions but does not affect their shape.

Figure A5: **Diffusion Predicted and Observed Cumulative Probability Distributions of Absolute Advantage in 1987**



Source: WTF (Feenstra et al. 2005, updated through 2008) for 133 time-consistent industries in 90 countries from 1962-2007 and CEPII.org; OLS gravity measures of export capability (log absolute advantage) $k = \ln A$ from (6).
Note: The graphs show the observed and predicted frequency of industries (the cumulative probability $1 - F_A(a)$ times the total number of industries $I = 133$) on the vertical axis plotted against the level of absolute advantage a (such that $A_{ist} \geq a$) on the horizontal axis, for the year $t = 1987$. Both axes have a log scale. The predicted frequencies are based on the GMM estimates of the comparative advantage diffusion (17) in Table 2 (parameters η and ϕ in column 1) and the inferred country-specific stochastic trend component $\ln Z_{st}$ from (18), which horizontally shifts the distributions but does not affect their shape.

Figure A6: **Diffusion Predicted and Observed Cumulative Probability Distributions of Absolute Advantage in 2007**



Source: WTF (Feenstra et al. 2005, updated through 2008) for 133 time-consistent industries in 90 countries from 1962-2007 and CEPII.org; OLS gravity measures of export capability (log absolute advantage) $k = \ln A$ from (6).
Note: The graphs show the observed and predicted frequency of industries (the cumulative probability $1 - F_A(a)$ times the total number of industries $I = 133$) on the vertical axis plotted against the level of absolute advantage a (such that $A_{ist} \geq a$) on the horizontal axis. Both axes have a log scale. The predicted frequencies are based on the GMM estimates of the comparative advantage diffusion (17) in Table 2 (parameters η and ϕ in column 1) and the inferred country-specific stochastic trend component $\ln Z_{st}$ from (18), which horizontally shifts the distributions but does not affect their shape.

E.3 Comparative advantage at varying industry aggregates

As a robustness check, we restrict the sample to the period 1984-2007 with industry aggregates from the SITC revision 2 classification. Data in this late period allow us to construct varying industry aggregates. We first obtain gravity-based estimates of log absolute advantage from OLS (6) at the refined industry aggregates. Following our benchmark specifications in the text, we then estimate the decay regression (10) at ten-year intervals and the GLD model (C.17) using GMM at five-year intervals.

For the decay regression, **Table A1** repeats in columns 1, 4 and 7 the estimates from **Table 1** for our benchmark sector aggregates at the SITC 2-3 digit level (133 industries) during the full sample period 1962-2007. **Table A1** presents in the remaining columns estimates for the SITC revision 2 two-digit level (60 industries) and the three-digit level (224 industries) during the late period 1984-2007. At the two-digit level (60 industries), the ten-year decay rate for absolute advantage using all countries and industries is -0.26, at the three-digit level (224 industries) it is -0.37. When using PPML-based log absolute advantage or the log RCA index, decay rates vary less across aggregation levels, ranging from -0.31 at the two-digit level for PPML-based log absolute advantage to -0.34 at the three-digit level for log RCA. The qualitative similarity in decay rates across definitions of export advantage and levels of industry aggregation suggest that our results are neither the byproduct of sampling error nor the consequence of industry definitions.

For the GLD model under the GMM procedure, Table A2 confirms that results remain largely in line with those in **Table 2** before, for the benchmark aggregates at the SITC 2-3 digit level (133 industries) during 1962-2007. The benchmark estimates are repeated in columns 1, 4 and 7. In the other columns, **Table A2** presents estimates for the SITC revision 2 two-digit level (60 industries) and the three-digit level (224 industries) during the late period 1984-2007.

Estimates of the dissipation rate η are slightly larger during the post-1984 period than over the full sample period and, similar to the implied η estimate in the decay regressions above, become smaller as we move from broader to finer classifications of industries. Estimates of the elasticity of decay ϕ are statistically significantly negative across all industry aggregates for the OLS-based absolute advantage measures but statistically indistinguishable from zero for PPML-based log absolute advantage and the log RCA index, again regardless of industry aggregation.

Table A1: DECAY REGRESSIONS FOR COMPARATIVE ADVANTAGE, VARYING INDUSTRY AGGREGATES

	OLS gravity k			PPML gravity k			$\ln RCA$		
		2-dgt.	3-dgt.		2-dgt.	3-dgt.		2-dgt.	3-dgt.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Decay Regression Coefficients									
Decay rate ρ	-0.349	-0.257	-0.370	-0.320	-0.320	-0.343	-0.303	-0.307	-0.326
	(0.002)***	(0.003)***	(0.002)***	(0.0002)***	(0.0003)***	(0.0003)***	(0.01)***	(0.017)***	(0.01)***
Var. of residual s^2	2.089	1.463	2.005	2.709	1.889	2.583	2.318	1.678	2.267
	(0.024)***	(0.027)***	(0.023)***	(0.013)***	(0.024)***	(0.017)***	(0.006)***	(0.009)***	(0.007)***
Implied Ornstein-Uhlenbeck (OU) Parameters									
Dissipation rate η	0.276	0.306	0.301	0.198	0.284	0.220	0.222	0.310	0.241
	(0.003)***	(0.006)***	(0.004)***	(0.0009)***	(0.004)***	(0.001)***	(0.006)***	(0.014)***	(0.006)***
Intensity of innovations σ	0.558	0.441	0.554	0.623	0.52	0.618	0.570	0.486	0.572
	(0.003)***	(0.004)***	(0.003)***	(0.001)***	(0.003)***	(0.002)***	(0.005)***	(0.008)***	(0.006)***
Observations	324,978	70,609	230,395	320,310	70,457	227,061	324,983	70,609	230,396
Adjusted R^2 (within)	0.222	0.241	0.265	0.282	0.315	0.295	0.216	0.233	0.224
Years t	36	14	14	36	14	14	36	14	14
Industries i	133	60	224	133	60	224	133	60	224
Source countries s	90	90	90	90	90	90	90	90	90

Source: WTF (Feenstra et al. 2005, updated through 2008) in 90 countries for 133 time-consistent industries from 1962-2007, for 60 time-consistent industries at the 2-digit SITC level from 1984-2007, and 224 industries at at the 3-digit SITC level from 1984-2007, and CEPII.org; OLS and PPML gravity measures of export capability (log absolute advantage) $k = \ln A$ from (6) and (8).

Note: Reported figures for ten-year changes. Variables are OLS and PPML gravity measures of log absolute advantage $\ln A_{ist}$ and the log Balassa index of revealed comparative advantage $\ln RCA_{ist} = \ln(X_{ist}/\sum_{\varsigma} X_{i\varsigma t})/(\sum_j X_{jst}/\sum_j \sum_{\varsigma} X_{j\varsigma t})$. OLS estimation of the ten-year decay rate ρ from

$$k_{is,t+10} - k_{ist} = \rho\, k_{ist} + \delta_{it} + \delta_{st} + \epsilon_{is,t+10},$$

conditional on industry-year and source country-year effects δ_{it} and δ_{st} for 1962-2007 (column 1-2) and 1984-2007 (columns 3-6). The implied dissipation rate η and squared innovation intensity σ^2 are based on the decay rate estimate ρ and the estimated variance of the decay regression residual $\hat{s}^2$ by (13). Robust standard errors, clustered at the industry level and corrected for generated-regressor variation of export capability k, for ρ and s^2, applying the multivariate delta method to standard errors for η and σ. * marks significance at ten, ** at five, and *** at one-percent level.

Table A2: GMM Estimates of Comparative Advantage Diffusion, Varying Industry Aggregates

	OLS gravity k			PPML gravity k			$\ln RCA$		
		2-dgt.	3-dgt.		2-dgt.	3-dgt.		2-dgt.	3-dgt.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Estimated Generalized Logistic Diffusion Parameters									
Dissipation rate η	0.256	0.297	0.287	0.180	0.289	0.205	0.212	0.332	0.204
	(0.004)***	(0.014)***	(0.004)***	(0.006)***	(0.107)***	(0.053)***	(0.006)***	(0.02)***	(0.035)***
Intensity of innovations σ	0.739	0.558	0.715	0.767	0.613	0.798	0.713	0.574	0.678
	(0.01)***	(0.011)***	(0.006)***	(0.037)***	(0.454)	(0.336)**	(0.051)***	(0.09)***	(0.42)
Elasticity of decay ϕ	-0.041	-0.070	-0.023	-0.009	-0.035	0.024	0.006	-0.014	-0.008
	(0.017)**	(0.024)***	(0.01)**	(0.035)	(0.593)	(0.272)	(0.053)	(0.119)	(0.404)
Implied Parameters									
Log gen. gamma scale $\ln\hat{\theta}$	121.94	59.09	281.50	900.95	155.22	-239.56	-1,410.50	548.19	1,083.20
	(71.526)*	(31.021)*	(161.258)*	(4581.812)	(3570.434)	(3595.553)	(14980.320)	(6069.679)	(72277.940)
Log gen. gamma shape $\ln\kappa$	5.017	4.115	6.338	7.788	5.456	5.842	8.641	7.484	8.181
	(0.842)***	(0.724)***	(0.875)***	(8.062)	(33.387)	(22.563)	(17.289)	(17.421)	(107.014)
Mean/median ratio	8.203	6.597	6.087	16.897	6.222	10.293	10.256	4.643	12.085
Observations	392,850	96,989	322,860	389,290	96,828	319,140	392,860	96,989	322,860
Industry-source obs. $I \times S$	11,542	5,332	19,160	11,531	5,331	19,118	11,542	5,332	19,160
Root mean sq. forecast error	1.851	1.690	1.737	1.898	1.664	1.817	1.760	1.560	1.768
Min. GMM obj. ($\times$ 1,000)	3.27e-13	1.82e-12	9.14e-13	2.56e-12	3.53e-11	9.91e-12	6.79e-12	1.61e-10	6.01e-11

Source: WTF (Feenstra et al. 2005, updated through 2008) in 90 countries for 133 time-consistent industries from 1962-2007, for 60 time-consistent industries at the 2-digit SITC level from 1984-2007, and 224 industries at at the 3-digit SITC level from 1984-2007, and CEPII.org; OLS and PPML gravity measures of export capability (log absolute advantage) $k = \ln A$ from (6) and (8).
Note: GMM estimation at the five-year horizon for the generalized logistic diffusion of comparative advantage $\hat{A}_{is}(t)$,

$$\mathrm{d}\ln\hat{A}_{is}(t) = -\frac{\eta\sigma^2}{2}\frac{\hat{A}_{is}(t)^{\phi}-1}{\phi}\,\mathrm{d}t + \sigma\,\mathrm{d}W_{is}^{\hat{A}}(t)$$

using absolute advantage $A_{is}(t) = \hat{A}_{is}(t)Z_s(t)$ based on OLS and PPML gravity measures of export capability k from (6), and the Balassa index of revealed comparative advantage $RCA_{ist} = (X_{ist}/\sum_{\varsigma} X_{i\varsigma t})/(\sum_j X_{jst}/\sum_j\sum_{\varsigma} X_{j\varsigma t})$. Parameters η, σ, ϕ for 1962-2007 (column 1-2) and 1984-2007 (columns 3-6) are estimated under the constraints $\ln\eta, \ln\sigma^2 > -\infty$ for the mirror Pearson (1895) diffusion of (20), while concentrating out country-specific trends $Z_s(t)$. The implied parameters are inferred as $\hat{\theta} = (\phi^2/\eta)^{1/\phi}$, $\kappa = 1/\hat{\theta}^{\phi}$and the mean/median ratio is given by (A.10). Robust errors in parentheses (corrected for generated-regressor variation of export capability k): * marks significance at ten, ** at five, and *** at one-percent level. Standard errors of transformed and implied parameters are computed using the multivariate delta method.

Supplementary Material for

The Dynamics of Comparative Advantage

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June 28, 2016

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S.1 Classifications

Our empirical analysis requires a time-invariant definition of less developed countries (LDC) and industrialized countries (non-LDC). Given our data time span of more then four decades (1962-2007), we classify the 90 economies, for which we obtain export capability estimates, by their relative status over the entire sample period.

In our classification, there are 28 *non-LDC*: Australia, Austria, Belgium-Luxembourg, Canada, China Hong Kong SAR, Denmark, Finland, France, Germany, Greece,Ireland, Israel, Italy, Japan, Kuwait, Netherlands, New Zealand, Norway, Oman, Portugal, Saudi Arabia, Singapore, Spain, Sweden, Switzerland, Trinidad and Tobago, United Kingdom, United States.

The remaining 62 countries are *LDC*: Algeria, Argentina, Bolivia, Brazil, Bulgaria, Cameroon, Chile, China, Colombia, Costa Rica, Côte d'Ivoire, Cuba, Czech Rep., Dominican Rep., Ecuador, Egypt, El Salvador, Ethiopia, Ghana, Guatemala, Honduras, Hungary, India, Indonesia, Iran, Jamaica, Jordan, Kenya, Lebanon, Libya, Madagascar, Malaysia, Mauritius, Mexico, Morocco, Myanmar, Nicaragua, Nigeria, Pakistan, Panama, Paraguay, Peru, Philippines, Poland, Rep. Korea, Romania, Russian Federation, Senegal, South Africa, Sri Lanka, Syria, Taiwan, Thailand, Tunisia, Turkey, Uganda, United Rep. of Tanzania, Uruguay, Venezuela, Vietnam, Yugoslavia, Zambia.

We split the industries in our sample by broad sector. The manufacturing sector includes all industries with an SITC one-digit code between 5 and 8. The nonmanufacturing merchandise sector includes industries in the agricultural sector as well industries in the mining and extraction sectors and spans the SITC one-digit codes from 0 to 4.

S.2 Application of the GMM generated-variable correction to second-stage OLS estimation

To adapt the results in **Appendix D.2** to the decay regression, we need to specify the appropriate moment condition and to account for the use of export capability estimates, instead of treating absolute advantage or comparative advantage as data.

Consider the decay relationship (10) and suppose true export capability were observed. Then, for any time interval Δ such as ten years,

$$k_{is,t+\Delta} - k_{ist} = \rho k_{ist} + \delta_{it} + \delta_{st} + \epsilon_{is,t+\Delta}. \tag{S.1}$$

The OLS estimator for ρ and the residual variance s^2 is the GMM estimator for the following conditional moment

$$\mathbb{E}_{ist}\mathbf{g}(\theta, k_{is,t+\Delta}, k_{ist}; \boldsymbol{\delta}) \equiv \mathbb{E}_{ist}\begin{pmatrix} (k_{is,t+\Delta} - k_{ist} - \rho k_{ist} - \delta_{it} - \delta_{st})k_{ist} \\ s^2 - (k_{is,t+\Delta} - k_{ist} - \rho k_{ist} - \delta_{it} - \delta_{st})^2 \end{pmatrix} = \mathbf{0}, \tag{S.2}$$

where $\theta = (\rho, s^2)'$ and $\boldsymbol{\delta}$ collects the industry-year and country-year fixed effects. We do not calculate a correction for standard errors on the industry-time and country-time fixed effects.

In the decay regression, we work with estimates of export capability directly and only use time series pairs spaced exactly Δ years apart. Let $\mathcal{S}_{it}$ denote the set of countries exporting good i in year t and also export good i in year $t+\Delta$. The effective sample size is $N \equiv \sum_{t=1}^{I} \sum_{t=1}^{T-\Delta} |\mathcal{S}_{it}|$. Denote the OLS estimator of θ with $\hat{\theta}_N$ and the OLS estimator for δ with $\hat{\delta}_N$.

A mean value expansion of the GMM criterion function (S.2) evaluated at the export capability estimates and estimates of the fixed effects gives

$$\mathbf{g}(\hat{\theta}_N, k_{is,t+\Delta}^{\text{OLS}}, k_{ist}^{\text{OLS}}; \hat{\delta}_N) = \underbrace{\mathbf{g}\left(\theta_0, k_{is,t+\Delta}, k_{ist}; \delta_0\right)}_{\equiv \mathbf{G}_{ist}^0} + \underbrace{\frac{\partial}{\partial \theta}\mathbf{g}\left(\theta, \tilde{k}_{is,t+\Delta}, \tilde{k}_{ist}; \tilde{\delta}_N\right)\bigg|_{\theta=\tilde{\theta}_N}}_{\equiv \tilde{\mathbf{G}}_{ist}^1} (\hat{\theta}_N - \theta_0)$$
$$+ \underbrace{\frac{\partial}{\partial k^F}\mathbf{g}\left(\tilde{\theta}_N, k^F, \tilde{k}_{ist}; \tilde{\delta}_N\right)\bigg|_{k^F=\tilde{k}_{is,t+\Delta}}}_{\equiv \tilde{\mathbf{G}}_{ist}^2} \left(k_{is,t+\Delta}^{\text{OLS}} - k_{is,t+\Delta}\right) + \underbrace{\frac{\partial}{\partial k}\mathbf{g}\left(\tilde{\theta}_N, \tilde{k}_{ist}, k; \tilde{\delta}_N\right)\bigg|_{k=\tilde{k}_{ist}}}_{\equiv \tilde{\mathbf{G}}_{ist}^3} \left(k_{ist}^{\text{OLS}} - k_{ist}\right),$$

where $|\tilde{\theta}_N - \theta_0| \le |\hat{\theta}_N - \theta_0|$, $|\tilde{\delta}_N - \delta_0| \le |\hat{\delta}_N - \delta_0|$, and $|\tilde{k}_{ist} - k_{ist}| \le |k_{ist}^{\text{OLS}} - k_{ist}|$. From this mean-value expansion, we obtain

$$\sqrt{N}(\hat{\theta}_N - \theta_0) =$$
$$-\left[\tilde{\mathbf{\Lambda}}_N' \mathbf{W} \tilde{\mathbf{\Lambda}}_N\right]^{-1} \tilde{\mathbf{\Lambda}}_N' \mathbf{W} \frac{1}{\sqrt{N}} \sum_{i=1}^{I} \sum_{t=1}^{T-\Delta} \sum_{s \in \mathcal{S}_{it}} \frac{N}{I|\mathcal{S}_{it}|(T-\Delta)} \left[\mathbf{G}_{ist}^0 + \tilde{\mathbf{G}}_{ist}^2 (k_{is,t+\Delta}^{\text{OLS}} - k_{is,t+\Delta}) + \tilde{\mathbf{G}}_{ist}^3 (k_{ist}^{\text{OLS}} - k_{ist})\right]$$

where $\tilde{\mathbf{\Lambda}}_N = [1/I(T-\Delta)] \sum_{i=1}^{I} \sum_{t=1}^{T-\Delta} (1/|\mathcal{S}_{it}|) \sum_{s \in \mathcal{S}_{it}} \tilde{\mathbf{G}}_{ist}^1$.

The sum in this expression can be rewritten as

$$\frac{1}{\sqrt{N}} \sum_{i=1}^{I} \sum_{t=1}^{T-\Delta} \sum_{s \in \mathcal{S}_{it}} \frac{N}{I|\mathcal{S}_{it}|(T-\Delta)} \left[\mathbf{G}_{ist}^0 + \tilde{\mathbf{G}}_{ist}^2 (k_{is,t+\Delta}^{\text{OLS}} - k_{is,t+\Delta}) + \tilde{\mathbf{G}}_{ist}^3 (k_{ist}^{\text{OLS}} - k_{ist})\right]$$
$$= \frac{1}{\sqrt{N}} \sum_{i=1}^{I} \sum_{t=1}^{T-\Delta} \sum_{s \in \mathcal{S}_{it}} \frac{N}{I|\mathcal{S}_{it}|(T-\Delta)} \mathbf{G}_{ist}^0 + o_p(1)$$
$$+ \frac{1}{\sqrt{N}} \sum_{t=1}^{T} \sum_{i=1}^{I} \underbrace{\sum_{s=1}^{S} \left[\mathbf{1}\{s \in \mathcal{S}_{i,t-\Delta}\} \frac{N}{I|\mathcal{S}_{i,t-\Delta}|(T-\Delta)} \mathbf{G}_{is,t-\Delta}^2 + \mathbf{1}\{s \in \mathcal{S}_{it}\} \frac{N}{I|\mathcal{S}_{it}|(T-\Delta)} \mathbf{G}_{ist}^3\right] (k_{ist}^{\text{OLS}} - k_{ist})}_{\equiv \mathbf{L}_{it}}$$

given that

$$\tilde{\mathbf{G}}_{ist}^2 \xrightarrow{p} \mathbf{G}_{ist}^2 \equiv \frac{\partial}{\partial k^F}\mathbf{g}\left(\theta_0, k^F, k_{ist}; \delta_0\right)\bigg|_{k^F=k_{is,t+\Delta}},$$
$$\tilde{\mathbf{G}}_{ist}^3 \xrightarrow{p} \mathbf{G}_{ist}^3 \equiv \frac{\partial}{\partial k}\mathbf{g}\left(\theta_0, k_{is,t+\Delta}, k; \delta_0\right)\bigg|_{k=k_{ist}}.$$

Define the matrix $\mathbf{G}_{it}$ so that its s'th column is

$$[\mathbf{G}_{it}]_{\cdot s} \equiv \left[\mathbf{1}\{s \in \mathcal{S}_{i,t-\Delta}\} \frac{N}{I|\mathcal{S}_{i,t-\Delta}|(T-\Delta)} \mathbf{G}_{is,t-\Delta}^2 + \mathbf{1}\{s \in \mathcal{S}_{it}\} \frac{N}{I|\mathcal{S}_{it}|(T-\Delta)} \mathbf{G}_{ist}^3\right]. \tag{S.3}$$

Then the vector $\mathbf{L}_{it}$ is

$$\mathbf{L}_{it} = \mathbf{G}_{it}(\mathbf{k}_{i\cdot t}^{\text{OLS}} - \mathbf{k}_{i\cdot t}).$$

Based on these derivations, the following proposition states the corrected asymptotic distribution for the coefficients in the decay regression.

Proposition 5. *Under the conditions of Proposition 3 and Assumptions 1, 2, and 3 we have that*

$$\sqrt{N}(\hat{\boldsymbol{\theta}}_N - \boldsymbol{\theta}_0) \xrightarrow{d} \mathcal{N}(\mathbf{0}, (\boldsymbol{\Lambda}'\mathbf{W}\boldsymbol{\Lambda})^{-1}\boldsymbol{\Lambda}'\mathbf{W}(\boldsymbol{\Xi} + \boldsymbol{\Omega})\mathbf{W}\boldsymbol{\Lambda}(\boldsymbol{\Lambda}'\mathbf{W}\boldsymbol{\Lambda})^{-1})$$

with

$$\begin{aligned}
\boldsymbol{\Lambda} &\equiv \mathbb{E}\,\frac{\partial}{\partial\boldsymbol{\theta}}\mathbf{g}\left(\boldsymbol{\theta}_0, k_{is,t+\Delta}, k_{ist}; \boldsymbol{\delta}_0\right), \\
\boldsymbol{\Xi} &\equiv \mathbb{E}\,\mathbf{g}\left(\boldsymbol{\theta}_0, k_{is,t+\Delta}, k_{ist}; \boldsymbol{\delta}_0\right)\mathbf{g}\left(\boldsymbol{\theta}_0, k_{is,t+\Delta}, k_{ist}; \boldsymbol{\delta}_0\right)', \\
\boldsymbol{\Omega} &\equiv \lim_{N\to\infty}\frac{1}{ND}\sum_{i=1}^{I}\sum_{t=1}^{T}\mathbf{G}_{it}\boldsymbol{\Sigma}_{it}^{*}\mathbf{G}_{it}'/\omega_{it},
\end{aligned}$$

where the s'th column of the matrix $\mathbf{G}_{it}$ is defined as in (S.3), and ω_{it} and $\boldsymbol{\Sigma}_{it}^{*}$ are defined as in Appendix D.2.

Proof. The proof follows the same logic as the proof of Proposition 4, but uses the asymptotic expansion derived in this section. □

S.3 The variance-covariance matrix of η and σ^2

Consider mean reversion of export capability under (S.1):

$$k_{is,t+\Delta} - k_{ist} = \rho k_{ist} + \delta_{it} + \delta_{st} + \epsilon_{is,t+\Delta}.$$

The coefficient ρ measures the fraction of log comparative advantage that dissipates over the time interval Δ. A constant ρ implies that dissipation is symmetric in the sense that export capability below zero reverts towards zero at the same rate as export capability above zero.

Suppose an Ornstein-Uhlenbeck (OU) process generates log comparative advantage $\ln \hat{A}_{is}(t)$ in continuous time, consistent with mean reversion of export capability following (S.1):

$$\mathrm{d}\ln \hat{A}_{is}(t) = -\frac{\eta\sigma^2}{2}\ln \hat{A}_{is}(t)\,\mathrm{d}t + \sigma\,\mathrm{d}W_{is}^{\hat{A}}(t), \tag{S.4}$$

where $W_{is}^{\hat{A}}(t)$ is a Wiener process that induces stochastic innovations in comparative advantage.[51] Equation (S.4) simply restates (11) from the text.

The discrete-time process that results from sampling from an OU process at a fixed time interval Δ is a Gaussian first-order autoregressive process with autoregressive parameter $\exp\{-\eta\sigma^2\Delta/2\}$ and innovation variance $(1 - \exp\{-\eta\sigma^2\Delta\})/\eta$ (Aït-Sahalia et al. 2010, Example 13). Applying this insight to the first-difference equation (S.1), we obtain

$$\begin{aligned}
\rho &\equiv -(1 - \exp\{-\eta\sigma^2\Delta/2\}) < 0, \\
s^2 &= (1 - \exp\{-\eta\sigma^2\Delta\})/\eta > 0,
\end{aligned} \tag{S.5}$$

[51] Recall from definition (14) that comparative advantage in continuous time is $\hat{A}_{is}(t) \equiv A_{is}(t)/Z_s(t)$, where $A_{is}(t) = \exp\{k_{is}(t)\}/\exp\{(1/S)\sum_{\varsigma} k_{i\varsigma}(t)\}$ is measured absolute advantage by (7) and $Z_s(t)$ is an unobserved country-wide stochastic trend.

as also shown in the main text, where s^2 is the variance of the residual $\epsilon_{is}(t,t+\Delta)$ in (S.1) and the residual is normally distributed with mean zero. The decay model (S.1) is equivalent to an OU process with $\eta > 0$ given the unobserved country fixed effect $\delta_s(t) \equiv \ln Z_s(t+\Delta) - (1+\rho)\ln Z_s(t)$. An OU process with $\rho \in (-1,0)$ generates a log normal stationary distribution of absolute advantage $A_{is}(t) = Z_s(t)\hat{A}_{is}(t)$ in the cross section, with a shape parameter of $1/\eta$ and a mean of zero.

The two equations (S.5) in (ρ, s^2) can be solved out for the equivalent OU parameters (η, σ^2):

$$\begin{aligned} \eta &= \frac{1-(1+\rho)^2}{s^2} > 0, \\ \sigma^2 &= \frac{\ln(1+\rho)^{-2}}{\Delta\,\eta} = \frac{s^2}{1-(1+\rho)^2}\frac{\ln(1+\rho)^{-2}}{\Delta} > 0. \end{aligned} \tag{S.6}$$

To express derivations more compactly, we consider the OU parameter vector $(\eta, \sigma^2)'$ a function $\mathbf{h}(\rho, s^2; \Delta)$ with

$$\begin{pmatrix} \eta \\ \sigma^2 \end{pmatrix} = \mathbf{h}(\rho, s^2; \Delta) \equiv \begin{pmatrix} \frac{1-(1+\rho)^2}{s^2} \\ \frac{s^2}{1-(1+\rho)^2}\frac{\ln(1+\rho)^{-2}}{\Delta} \end{pmatrix}. \tag{S.7}$$

Estimation. The OU process implies that equation (S.1) satisfies the assumptions of the classic regression model. Estimation of (S.1) with ordinary least squares therefore provides us with consistent estimators:

$$\begin{aligned} (\hat{\rho}, \hat{\boldsymbol{\delta}}')' \equiv (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y} \quad &\sim \quad \mathcal{N}\left((\hat{\rho}, \hat{\boldsymbol{\delta}}')', s^2(\mathbf{X}'\mathbf{X})^{-1}\right) \\ \hat{s}^2 \equiv \frac{RSS}{N-P} \quad &\sim \quad \frac{s^2}{N-P}\,\chi^2_{N-P}, \end{aligned} \tag{S.8}$$

where $\mathbf{y} \equiv \mathbf{J}^S_{i,t+\Delta}\mathbf{k}_{i\cdot,t+\Delta} - \mathbf{J}^S_{it}\mathbf{k}_{i\cdot t}$ is the dependent variable, $\mathbf{X} \equiv [\mathbf{J}^S_{it}\mathbf{k}_{i\cdot t}, \mathbf{I}_{it}, \mathbf{I}_{st}]$ is the $N \times P$ matrix of regressors ($N \equiv \sum_{t=1}^{I}\sum_{t=1}^{T-\Delta}|\mathcal{S}_{it}|$), RSS is the residual sum of squares (the sum of the squared regression residuals), and χ^2_{N-P} denotes a χ^2-distributed variable with $N-P$ degrees of freedom.[52] The variance of the estimator $\hat{\rho}$ is $\mathbb{V}_{\hat{\rho}} = s^2(X'X)^{-1}$, the variance of the estimator $\hat{s}^2$ is $\mathbb{V}_{\hat{s}^2} = 2s^4/(N-P)$ by the χ^2-distribution, and the estimators $\hat{\rho}$ and $\hat{s}^2$ are independent of each other by the properties of the classic regression model. For convenience, we define the variance-covariance matrix between the two estimators as

$$\boldsymbol{\Sigma}_{\rho,s^2} \equiv \begin{pmatrix} \mathbf{V}_{\hat{\rho}} & 0 \\ 0 & \mathbf{V}_{\hat{s}^2} \end{pmatrix} = \begin{pmatrix} s^2(\mathbf{X}'\mathbf{X})^{-1} & 0 \\ 0 & 2s^4/(N-P) \end{pmatrix}. \tag{S.9}$$

By (S.6) and (S.7), the according estimators of the equivalent OU parameters $(\hat{\eta}, \hat{\sigma}^2)$ can be compactly written as the function

$$\begin{pmatrix} \hat{\eta} \\ \hat{\sigma}^2 \end{pmatrix} = \mathbf{h}(\hat{\rho}, \hat{s}^2; \Delta).$$

By the multivariate delta method, this estimator is normally distributed with

$$\begin{pmatrix} \hat{\eta} \\ \hat{\sigma}^2 \end{pmatrix} \sim \mathcal{N}\left(\mathbf{h}(\rho, s^2; \Delta), \nabla_{\{\rho,s^2\}}\mathbf{h}(\rho, s^2; \Delta) \cdot \boldsymbol{\Sigma}_{\rho,s^2} \cdot \nabla_{\{\rho,s^2\}}\mathbf{h}(\rho, s^2; \Delta)'\right), \tag{S.10}$$

[52] As in Appendix D, $\mathbf{k}_{i\cdot t}$ denotes the vector of export capabilities of industry i at time t across countries and $\mathbf{J}^S_{it}$ is a matrix of indicators reporting the exporter country by observation.

where

$$\nabla_{\{\rho,s^2\}}\mathbf{h}\big(\rho,s^2;\Delta\big)=\begin{pmatrix}\frac{\partial\eta}{\partial\rho} & \frac{\partial\eta}{\partial s^2}\\ \frac{\partial\sigma^2}{\partial\rho} & \frac{\partial\sigma^2}{\partial s^2}\end{pmatrix}=\begin{pmatrix}-\frac{2(1+\rho)}{s^2} & -\frac{1-(1+\rho)^2}{s^4}\\ -\frac{2s^2}{\Delta}\frac{\left[1-(1+\rho)^2\right]-(1+\rho)^2\ln(1+\rho)^{-2}}{(1+\rho)^2[1-(1+\rho)^2]^2} & \frac{\ln(1+\rho)^{-2}}{\Delta[1-(1+\rho)^2]}\end{pmatrix}$$

with $\partial\eta/\partial\rho, \partial\eta/\partial s^2, \partial\sigma^2/\partial\rho < 0$ and $\partial\sigma^2/\partial s^2 > 0$ for $\rho \in (-1,0)$. For clarity, using (S.9) the variance-covariance matrix of the estimator $(\hat\eta,\hat\sigma^2)'$ can also be rewritten as

$$\nabla_{\{\rho,s^2\}}\mathbf{h}\big(\rho,\hat s^2;\Delta\big)\cdot\mathbf{\Sigma}_{\rho,s^2}\cdot\nabla_{\{\rho,s^2\}}\mathbf{h}\big(\rho,s^2;\Delta\big)'=\begin{pmatrix}\left(\frac{\partial\eta}{\partial\rho}\right)^2\mathbf{V}_{\hat\rho}+\left(\frac{\partial\eta}{\partial s^2}\right)^2\mathbf{V}_{\hat s^2} & \frac{\partial\eta}{\partial\rho}\frac{\partial\sigma^2}{\partial\rho}\mathbf{V}_{\hat\rho}+\frac{\partial\eta}{\partial s^2}\frac{\partial\sigma^2}{\partial s^2}\mathbf{V}_{\hat s^2}\\ \frac{\partial\eta}{\partial\rho}\frac{\partial\sigma^2}{\partial\rho}\mathbf{V}_{\hat\rho}+\frac{\partial\eta}{\partial s^2}\frac{\partial\sigma^2}{\partial s^2}\mathbf{V}_{\hat s^2} & \left(\frac{\partial\sigma^2}{\partial\rho}\right)^2\mathbf{V}_{\hat\rho}+\left(\frac{\partial\sigma^2}{\partial s^2}\right)^2\mathbf{V}_{\hat s^2}\end{pmatrix}.$$

Similarly, for the full vector of all estimators $\mathbf{H}(\rho,s^2;\Delta)\equiv(\hat\rho,\hat s^2,\hat\eta,\hat\sigma^2)'$ the variance-covariance matrix $\mathbf{Cov}=\nabla_{\{\rho,s^2\}}\mathbf{H}\big(\rho,\hat s^2;\Delta\big)\cdot\mathbf{\Sigma}_{\rho,s^2}\cdot\nabla_{\{\rho,s^2\}}\mathbf{H}\big(\rho,s^2;\Delta\big)'$ can be written as

$$\mathbf{Cov}=\begin{pmatrix}\mathbf{V}_{\hat\rho} & 0 & \frac{\partial\eta}{\partial\rho}\mathbf{V}_{\hat\rho} & \frac{\partial\sigma^2}{\partial\rho}\mathbf{V}_{\hat\rho}\\ 0 & \mathbf{V}_{\hat s^2} & \frac{\partial\eta}{\partial s^2}\mathbf{V}_{\hat s^2} & \frac{\partial\sigma^2}{\partial s^2}\mathbf{V}_{\hat s^2}\\ \frac{\partial\eta}{\partial\rho}\mathbf{V}_{\hat\rho} & \frac{\partial\eta}{\partial s^2}\mathbf{V}_{\hat s^2} & \left(\frac{\partial\eta}{\partial\rho}\right)^2\mathbf{V}_{\hat\rho}+\left(\frac{\partial\eta}{\partial s^2}\right)^2\mathbf{V}_{\hat s^2} & \frac{\partial\eta}{\partial\rho}\frac{\partial\sigma^2}{\partial\rho}\mathbf{V}_{\hat\rho}+\frac{\partial\eta}{\partial s^2}\frac{\partial\sigma^2}{\partial s^2}\mathbf{V}_{\hat s^2}\\ \frac{\partial\sigma^2}{\partial\rho}\mathbf{V}_{\hat\rho} & \frac{\partial\sigma^2}{\partial s^2}\mathbf{V}_{\hat s^2} & \frac{\partial\eta}{\partial\rho}\frac{\partial\sigma^2}{\partial\rho}\mathbf{V}_{\hat\rho}+\frac{\partial\eta}{\partial s^2}\frac{\partial\sigma^2}{\partial s^2}\mathbf{V}_{\hat s^2} & \left(\frac{\partial\sigma^2}{\partial\rho}\right)^2\mathbf{V}_{\hat\rho}+\left(\frac{\partial\sigma^2}{\partial s^2}\right)^2\mathbf{V}_{\hat s^2}\end{pmatrix}.$$

S.4 Top products

Table S1 shows the top two products in terms of normalized log absolute advantage $\ln A_{ist}$ for 28 of the 90 exporting countries, using 1987 and 2007 as representative years. To obtain a measure of comparative advantage, we normalize log absolute advantage by its country mean: $\ln A_{ist} - (1/I)\sum_j^I \ln A_{jst}$. The country normalization of log absolute advantage $\ln A_{ist}$ results in a double log difference of export capability k_{ist}—a country's log deviation from the global industry mean in export capability less its average log deviation across all industries. For comparison, **Table S2** presents the top two products in terms of the Balassa RCA index.

S.5 Absolute advantage and export shares

To verify that our measure of export advantage (7) does not peg obscure industries as top sectors, we plot $\ln A_{ist}$ against the log of the share of the industry in national exports $\ln\Big(X_{ist}/(\sum_j X_{jst})\Big)$. As **Figure S1** documents for the years 1967, 1987 and 2007, there is a strongly positive correlation between log absolute advantage and the log industry share of national exports. This correlation is 0.77 in 1967, 0.78 in 1987, and 0.83 in 2007. (For comparison, the correlation between $\ln A_{ist}$ and the log Balassa RCA index in these same years is 0.69, 0.70, and 0.68, respectively.

S.6 Additional evidence on cumulative probability distributions

We repeat the cumulative probability distribution plots, which were based on OLS estimated gravity measures of absolute advantage, now using absolute advantage measures based on the Poisson pseudo-maximum-likelihood (PPML) gravity model proposed by Silva and Tenreyro (2006). This exercise helps us verify that the cross sectional distributions of OLS-based absolute advantage in **Figures A2** and **A3** in the Appendix to the main paper are robust to alternative gravity estimation that can accommodate zero bilateral trade flows. **Figures S2** and **S3** plot, for the same 28 countries in 1987 and 2007, the log number of a source country s's industries that

Table S1: **Top Two Industries by Normalized Absolute Advantage**

Country	1987		2007		Country	1987		2007	
Argentina	Maize, unmilled	4.22	Maize, unmilled	5.48	Mexico	Sulphur	3.63	Alcoholic beverages	4.00
	Animal feed	3.88	Oil seed	4.59		Other crude minerals	3.22	Office machines	3.85
Australia	Wool	3.74	Cheese and curd	3.23	Peru	Metal ores & concntr.	4.21	Metal ores & concntr.	6.33
	Jute	3.78	Fresh meat	3.18		Animal feed	4.00	Coffee	4.69
Brazil	Iron ore	3.61	Iron ore	5.18	Philippines	Vegetable oils & fats	3.85	Office machines	4.48
	Coffee	3.39	Fresh meat	4.47		Preserved fruits & nuts	3.54	Electric machinery	3.59
Canada	Sulphur	3.99	Wheat, unmilled	5.16	Poland	Barley, unmilled	5.34	Furniture	2.19
	Iron ore	3.57	Sulphur	3.34		Sulphur	3.26	Glassware	2.77
China	Explosives	7.33	Sound/video recorders	4.99	Rep. Korea	Radio receivers	5.57	Television receivers	6.06
	Maize, unmilled	6.89	Radio receivers	4.71		Television receivers	5.43	Telecomm. equipmt.	5.11
Czech Rep.	Glassware	4.06	Glassware	4.26	Romania	Furniture	3.55	Footwear	3.50
	Prep. cereal & flour	3.69	Road vehicles	3.67		Fertilizers, manuf.	2.73	Silk	3.16
Egypt	Cotton	4.46	Fertilizers, crude	4.34	Russia	Maize, unmilled	5.63	Animal oils & fats	8.11
	Textile yarn, fabrics	2.84	Rice	3.79		Pulp & waste paper	5.04	Fertilizers, manuf.	4.34
France	Electric machinery	3.52	Other transport eqpmt.	3.42	South Africa	Stone, sand & gravel	3.90	Iron & steel	4.17
	Alcoholic beverages	3.47	Alcoholic beverages	3.26		Radioactive material	3.62	Fresh fruits & nuts	3.47
Germany	Road vehicles	4.08	Metalworking mach.	2.78	Taiwan	Explosives	4.74	Television receivers	5.24
	General machinery	4.02	Meters & counters	0.75		Footwear	4.45	Office machines	5.06
Hungary	Margarine	3.21	Telecomm. equipmt.	4.21	Thailand	Rice	4.92	Rice	4.99
	Fresh meat	2.79	Office machines	4.14		Fresh vegetables	4.18	Natural rubber	4.57
India	Tea	4.23	Precious stones	3.89	Turkey	Fresh vegetables	3.45	Glassware	3.35
	Leather	3.92	Rice	3.65		Tobacco, unmanuf.	3.38	Textile yarn, fabrics	3.25
Indonesia	Natural rubber	5.02	Natural rubber	5.24	United States	Office machines	4.00	Other transport eqpmt.	3.49
	Improved wood	4.66	Sound/video recorders	4.88		Other transport eqpmt.	3.29	Photographic supplies	2.63
Japan	Sound/video recorders	6.37	Sound/video recorders	5.97	United Kingd.	Measuring instrmnts.	3.22	Alcoholic beverages	3.30
	Road vehicles	6.17	Road vehicles	5.70		Office machines	3.17	Pharmaceutical prod.	3.16
Malaysia	Natural rubber	6.19	Radio receivers	5.77	Vietnam	Maize, unmilled	7.63	Animal oils & fats	10.26
	Vegetable oils & fats	4.85	Sound/video recorders	5.01		Jute	5.16	Footwear	6.97

Source: WTF (Feenstra et al. 2005, updated through 2008) for 133 time-consistent industries in 90 countries from 1962-2007.
Note: Top two industries for 28 of the 90 countries in 1987 and 2007 in terms of normalized log absolute advantage, relative to the country mean: $\ln A_{ist} - (1/I) \sum_{i'}^{I} \ln A_{i'st}$.

Table S2: **Top Two Industries by Balassa Comparative Advantage**

Country	1987		2007		Country	1987		2007	
Argentina	Cereals, unmilled	3.65	Animal feed	3.74	Mexico	Stone, sand & gravel	2.14	Television receivers	2.20
	Dyeing extracts	3.20	Oil seed	3.34		Sulphur	2.12	Fresh vegetables	1.45
Australia	Wool	3.74	Uranium	4.57	Peru	Metal ores & concntr.	3.79	Animal oils & fats	4.07
	Uranium	3.58	Wool	4.04		Ores, precious metals	3.19	Metal ores & concntr.	3.83
Brazil	Iron ore	3.34	Iron ore	5.18	Philippines	Vegetable oils & fats	3.81	Office machines	4.41
	Preserved fruits/nuts	2.64	Tobacco, unmanuf.	3.21		Pres. fruits & nuts	3.50	Electric machinery	3.51
Canada	Sulphur	2.24	Cinemat. film, exposed	2.64	Poland	Sulphur	3.78	Smoked fish	2.19
	Pulp & waste paper	1.90	Sulphur	2.45		Preserved meat	2.66	Wood manuf.	1.71
China	Silk	3.77	Silk	1.97	Rep. Korea	Travel goods	1.88	Optical instrmnts.	2.15
	Jute	3.30	Travel goods	1.43		Footwear	1.78	Synthetic fibres	1.49
Czech Rep.	Glassware	2.03	Television receivers	1.61	Romania	Jute	3.28	Leather manuf.	3.03
	Metalworking mach.	1.74	Glassware	1.33		Fertilizers, manuf.	2.71	Silk	2.03
Egypt	Cotton	4.47	Fertilizers, crude	3.70	Russia	Ferrous scrap metal	2.58	Fertilizers, manuf.	3.00
	Vegetable fibres	2.57	Vegetable fibres	3.63		Raw furskins	5.02	Radioactive material	2.72
France	Alcoholic beverages	1.70	Vegetable fibres	2.21	South Africa	Uranium	3.92	Ores, precious metals	3.09
	Radioactive material	1.49	Alcoholic beverages	1.72		Ores, precious metals	3.02	Natural abrasives	2.71
Germany	Synthetic dye	0.95	Other man-made fibres	0.99	Taiwan	Travel goods	2.09	Optical instrmnts.	2.27
	Dyeing extracts	0.77	Meters & counters	0.75		Footwear	1.96	Synthetic fibres	1.51
Hungary	Preserved meat	2.22	Television receivers	1.63	Thailand	Rice	3.93	Natural rubber	3.24
	Crude animal materials	2.17	Maize, unmilled	1.58		Cereal meals & flour	3.31	Rice	3.03
India	Tea	3.81	Iron ore	2.68	Turkey	Tobacco unmanuf.	3.17	Tobacco unmanuf.	2.28
	Spices	3.10	Precious stones	2.65		Crude minerals	2.45	Lime, cement	2.11
Indonesia	Improved wood	3.90	Natural rubber	3.57	United States	Maize, unmilled	1.42	Maize, unmilled	1.64
	Natural rubber	3.88	Vegetable oils & fats	3.03		Oil seed	1.39	Cotton	1.56
Japan	Sound/video recorders	1.76	Photographic supplies	1.36	United Kingd.	Cinemat. film, exposed	1.47	Alcoholic beverages	1.27
	Photographic eqpmnt.	1.33	Photographic eqpmnt.	1.23		Precious stones	1.41	Rags	1.07
Malaysia	Natural rubber	3.90	Proc. animal/plant oils	2.71	Vietnam	Fresh shellfish	4.59	Rice	3.74
	Proc. animal/plant oils	3.56	Vegetable oils & fats	2.36		Spices	3.42	Natural rubber	3.12

Source: WTF (Feenstra et al. 2005, updated through 2008) for 133 time-consistent industries in 90 countries from 1962-2007.
Note: Top two industries for 28 of the 90 countries in 1987 and 2007 in terms of log revealed comparative advantage, using the log Balassa (1965) index $\ln RCA_{ist} = \ln(X_{ist}/\sum_{\varsigma} X_{i\varsigma t})/(\sum_j X_{jst}/\sum_j \sum_{\varsigma} X_{j\varsigma t})$.

Figure S1: **Absolute Advantage and Export Shares**

Source: WTF (Feenstra et al. 2005, updated through 2008) for 133 time-consistent industries in 90 countries from 1962-2007.
Note: The vertical axis shows a country-industry's gravity-based measure of log absolute advantage $\ln A_{ist}$ given by (7), the horizontal axis plots the same country-industry's share of the industry i's global export value: $X_{ist}/(\sum_{\varsigma} X_{i\varsigma t})$.

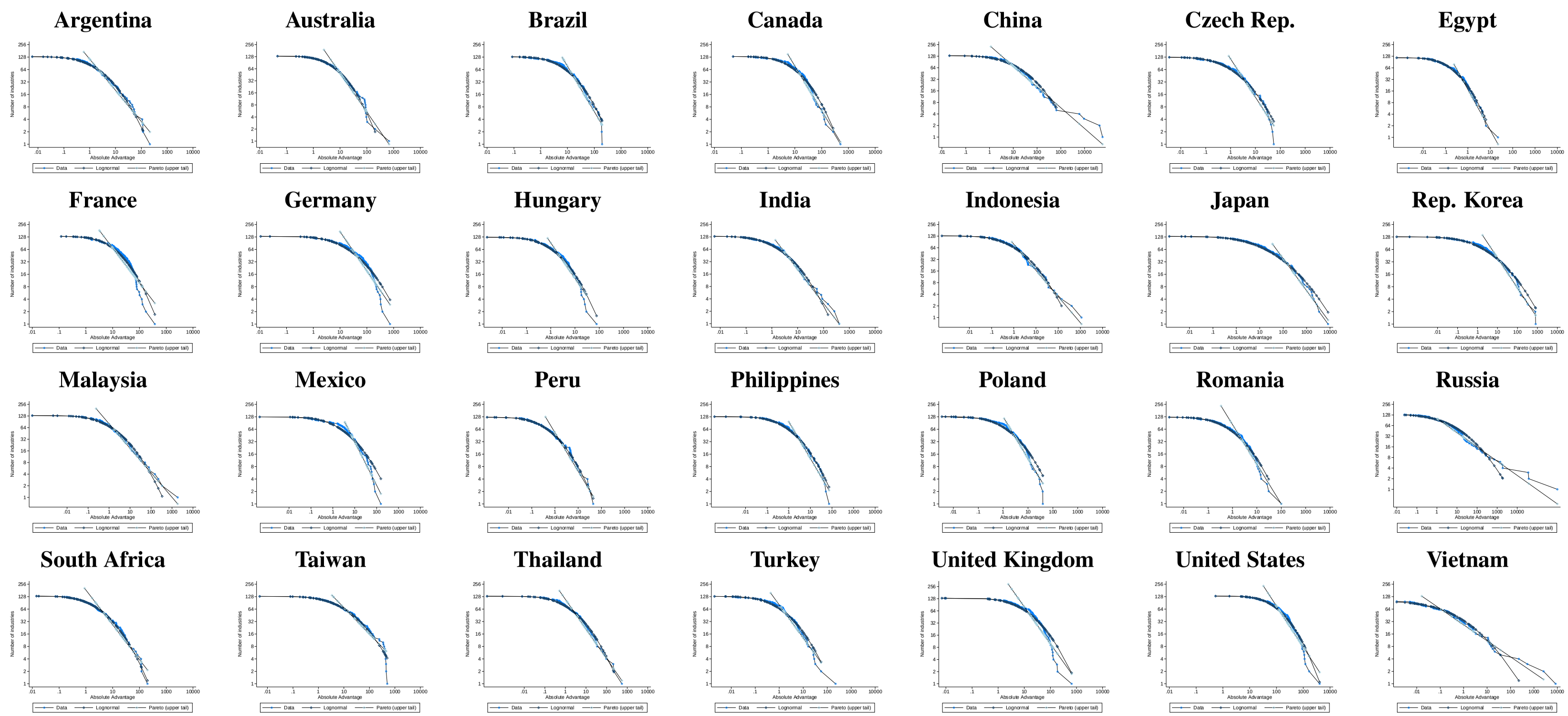
have at least a given level of PPML-based absolute advantage in year t against that comparative advantage level for industries i. The figures also graph the fit of the revealed comparative advantage index in the cross section to a log normal distribution using maximum likelihood separately for each country in each year. Results resemble those for export capabilities (7).

To verify that the graphed cross sectional distributions in **Figures A2** and **A3** in the Appendix to the main paper are not a byproduct of specification error in estimating export capabilities (7) from the gravity model, we also repeat the cumulative probability distribution plots using the revealed comparative advantage index by Balassa (1965) for comparative advantage. **Figures S4** and **S5** plot, for the same 28 countries in 1987 and 2007, the log number of a source country s's industries that have at least a given level of revealed comparative advantage $(X_{is}/\sum_{\varsigma} X_{i\varsigma})/(\sum_j X_{js}/\sum_j \sum_{\varsigma} X_{j\varsigma})$ in year t against that comparative advantage level for industries i. The figures also graph the fit of the revealed comparative advantage index in the cross section to a log normal distribution using maximum likelihood separately for each country in each year. Results broadly resemble those for export capabilities (7).

To verify that the graphed cross sectional distributions in **Figures A2** and **A3** in the Appendix are not a consequence of arbitrary industry aggregation, we construct plots also at the 2-digit and 3-digit levels, based on SITC revision 2 data in 1987 and 2007. The figures also graph the fit of log absolute advantage in the cross section to a log normal distribution using maximum likelihood separately for each country in each year. As **Figures S6** and S7 show for 60 time-consistent 2-digit industries, and **Figures S8** and S9 for 224 time-consistent 3-digit industries, stability across countries and over time in the curvatures are broadly similar to those for export capabilities at our benchmark SITC 2-3 digit level for 133 industries.

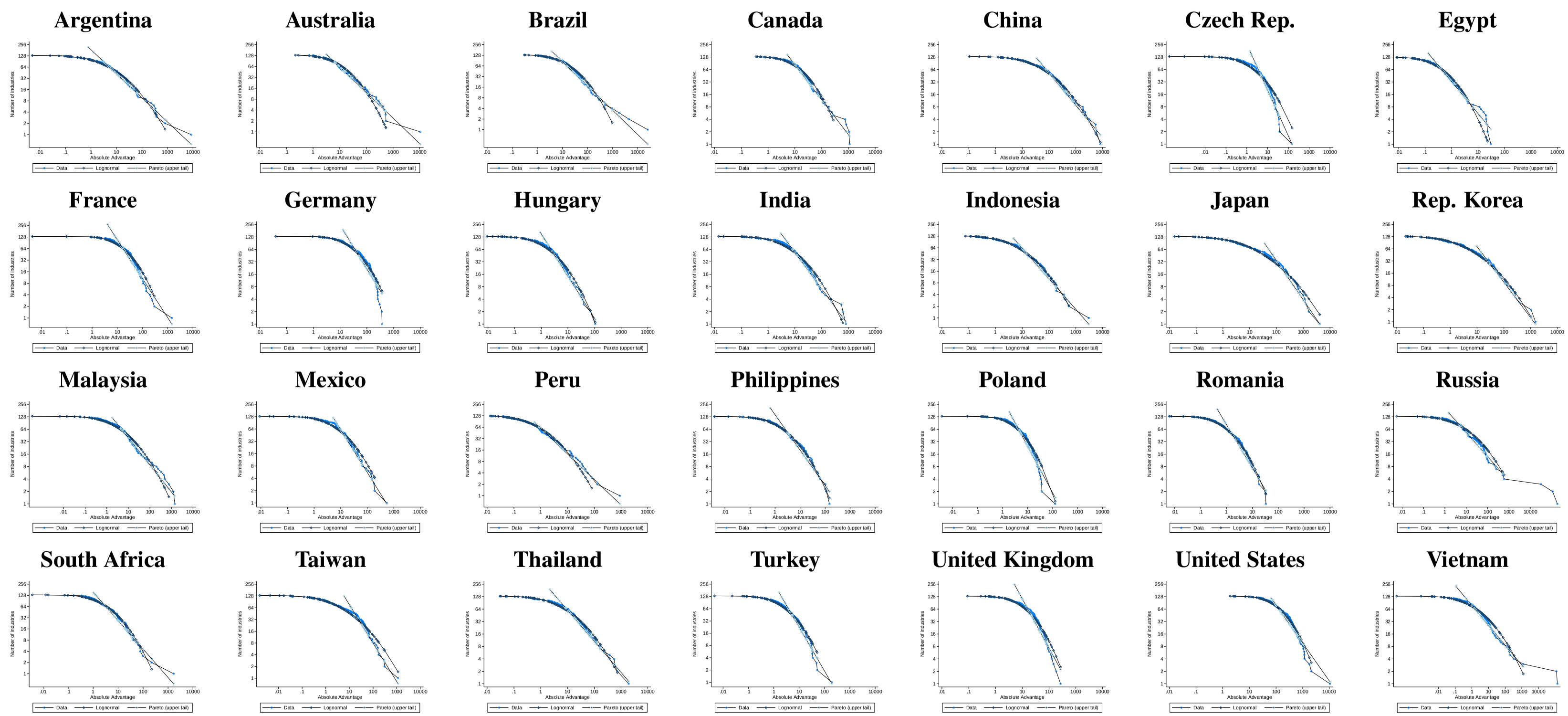
To further substantiate the stationarity of comparative advantage measures, we compare the pooled industry-level measures of comparative advantage across countries from OLS-based export capability to those from PPML-based export capability in **Figure S10**. We obtain log comparative advantage as the residuals from OLS projections on industry-year and source country-year effects and plot the percentiles of the global distribution of these comparative advantage measures over time. The time lines for the 5th/95th, 20th/80th, 30th/70th, and 45th/55th percentiles are, with minor fluctuation, parallel to the horizontal axis for both OLS-based and PPML-based export capability—a strong indication that the global distribution of comparative advantage is stationary. **Figure S11** plots a selection of time lines from **Figure S10** and compares them visually to the GMM estimates of the comparative advantage diffusion (17) in Table 2 (parameters η and ϕ in columns 1 and 4). The fit is close, especially for the PPML-based measures of export capability.

Figure S2: **Cumulative Probability Distribution of PPML Absolute Advantage for 28 Countries in 1987**



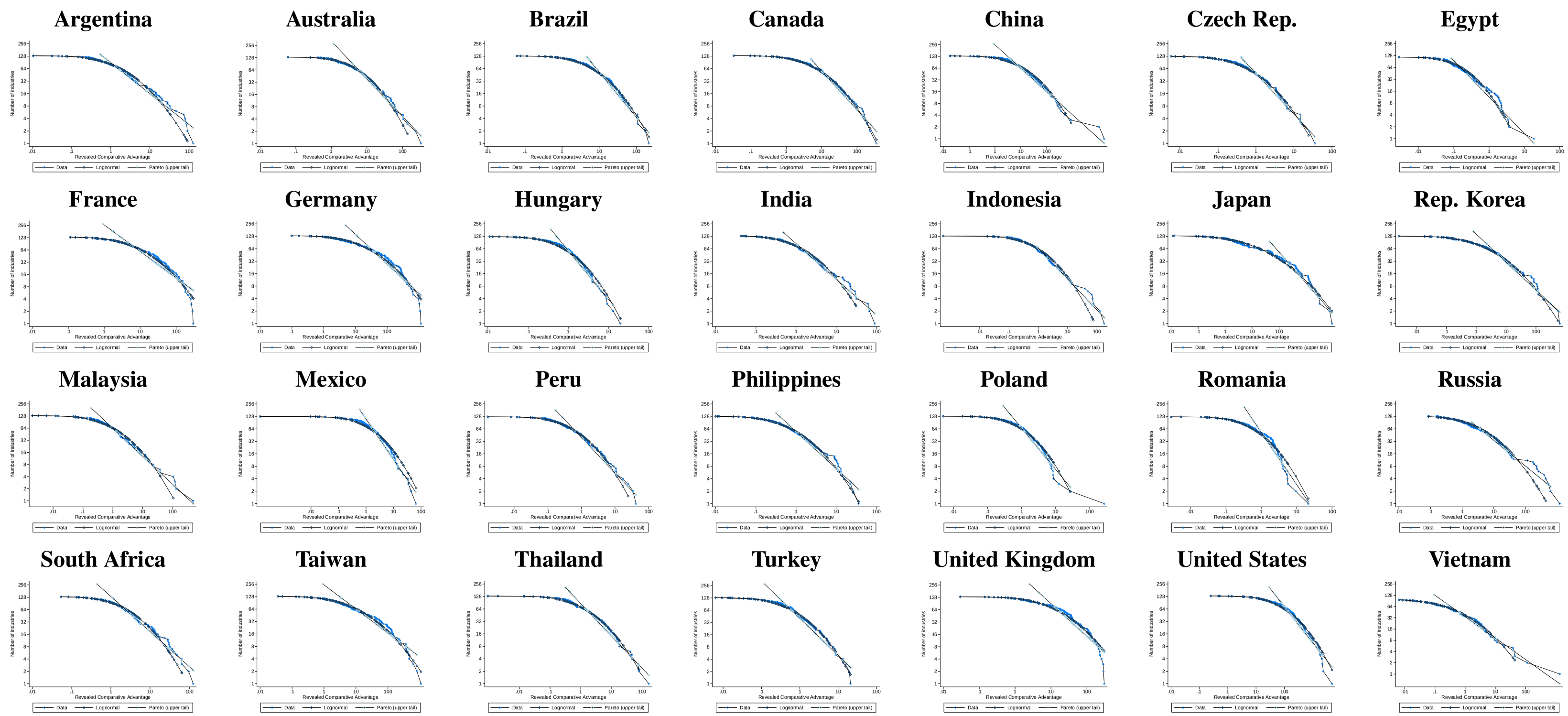
Source: WTF (Feenstra et al. 2005, updated through 2008) for 133 time-consistent industries in 90 countries from 1985-1987 and CEPII.org; three-year means of PPML gravity measures of export capability (log absolute advantage) $k = \ln A$ from (8).
Note: The graphs show the observed and predicted frequency of industries (the cumulative probability $1 - F_A(a)$ times the total number of industries $I = 133$) on the vertical axis plotted against the level of absolute advantage a (such that $A_{ist} \geq a$) on the horizontal axis, for the year $t = 1987$. Both axes have a log scale.

Figure S3: **Cumulative Probability Distribution of PPML Absolute Advantage for 28 Countries in 2007**



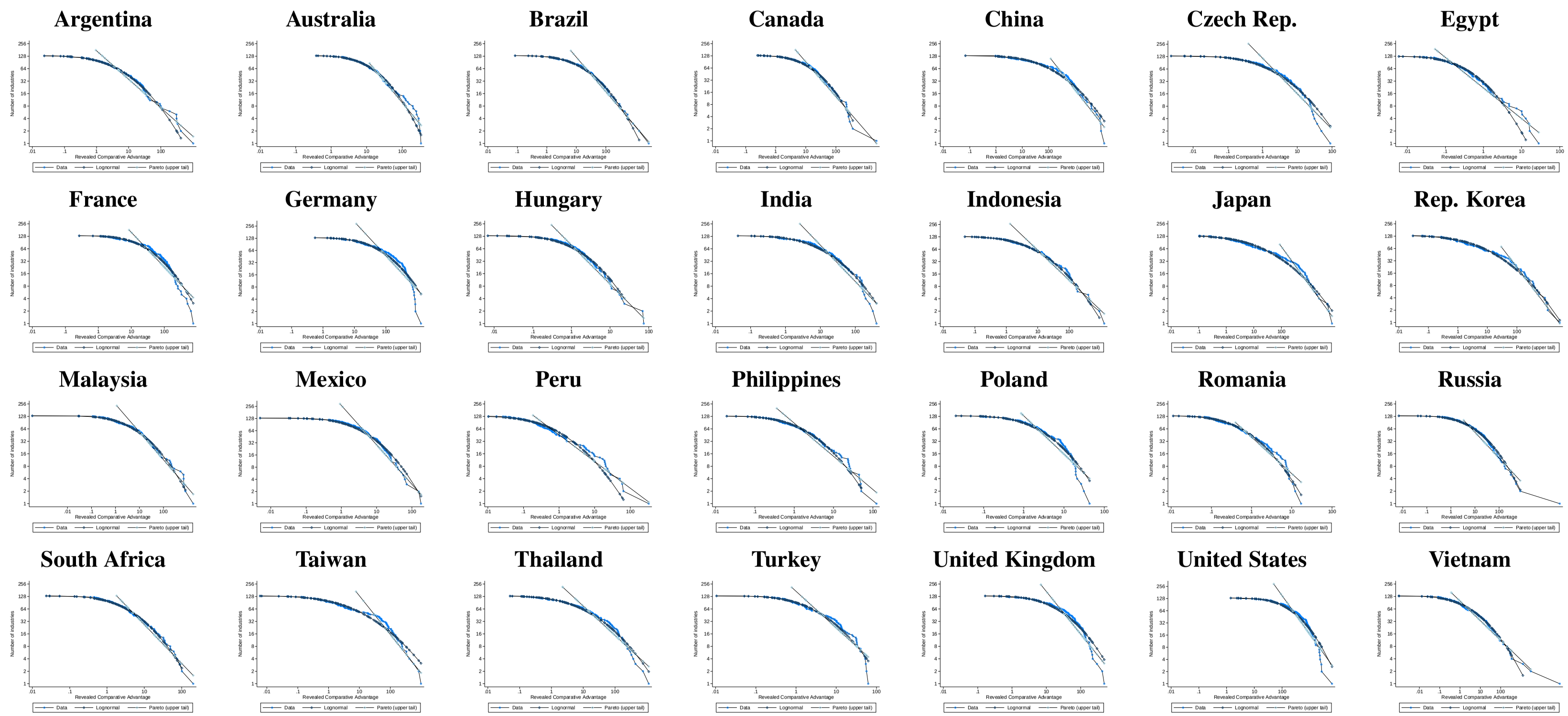
Source: WTF (Feenstra et al. 2005, updated through 2008) for 133 time-consistent industries in 90 countries from 2005-2007 and CEPII.org; three-year means of PPML gravity measures of export capability (log absolute advantage) $k = \ln A$ from (8).
Note: The graphs show the observed and predicted frequency of industries (the cumulative probability $1 - F_A(a)$ times the total number of industries $I = 133$) on the vertical axis plotted against the level of absolute advantage a (such that $A_{ist} \geq a$) on the horizontal axis, for the year $t = 2007$. Both axes have a log scale.

Figure S4: **Cumulative Probability Distribution of Balassa Revealed Comparative Advantage for 28 Countries in 1987**



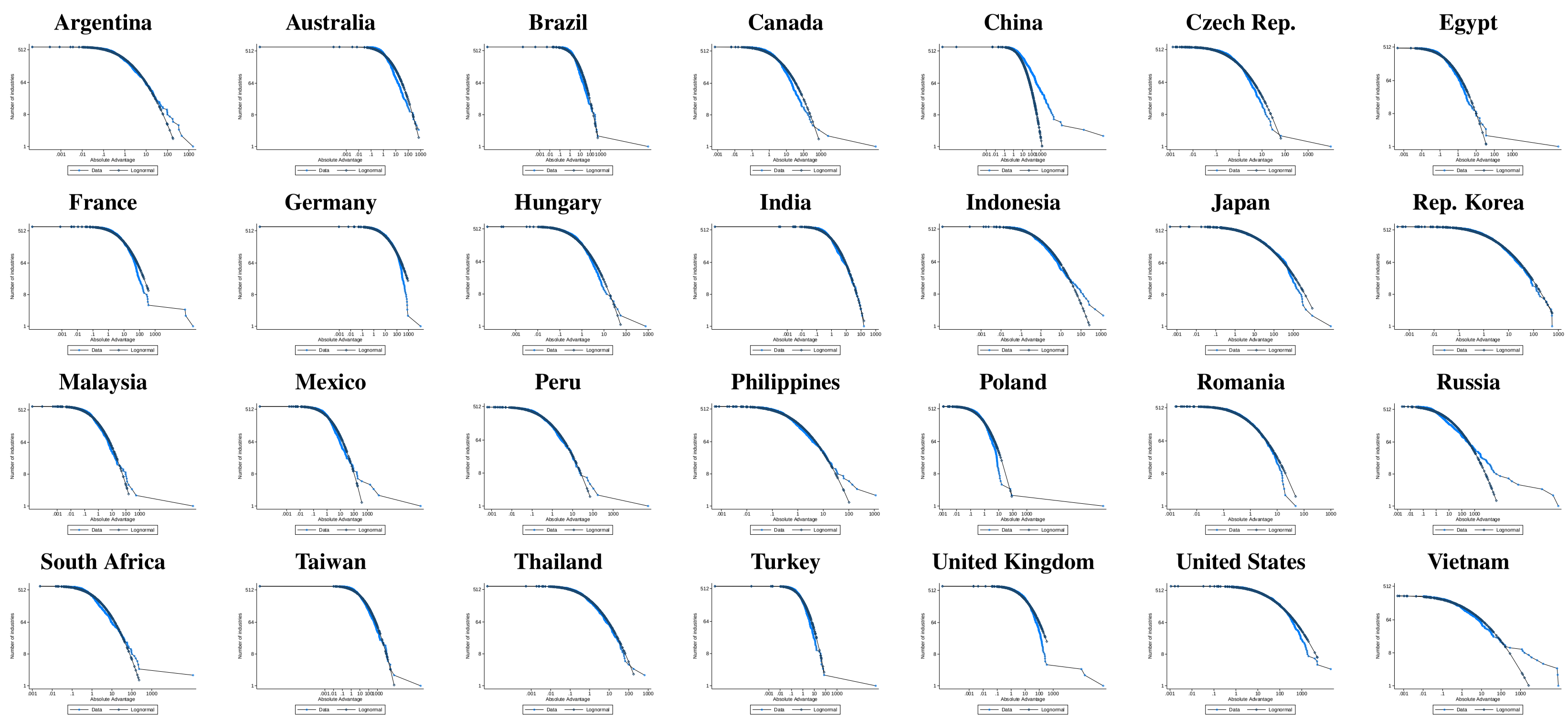
Source: WTF (Feenstra et al. 2005, updated through 2008) for 133 time-consistent industries in 90 countries from 1985-1987, three-year means of .
Note: The graphs show the frequency of industries (the cumulative probability $1 - F_{\hat{X}}(\hat{x})$ times the total number of industries $I = 133$) on the vertical axis plotted against the Balassa index of revealed comparative advantage $\hat{X} = (X_{is}/\sum_{\varsigma} X_{i\varsigma})/(\sum_j X_{js}/\sum_j \sum_{\varsigma} X_{j\varsigma})$ on the horizontal axis. Both axes have a log scale. The fitted log normal distribution is based on maximum likelihood estimation by country s in year $t = 1987$.

Figure S5: **Cumulative Probability Distribution of Balassa Revealed Comparative Advantage for 28 Countries in 2007**



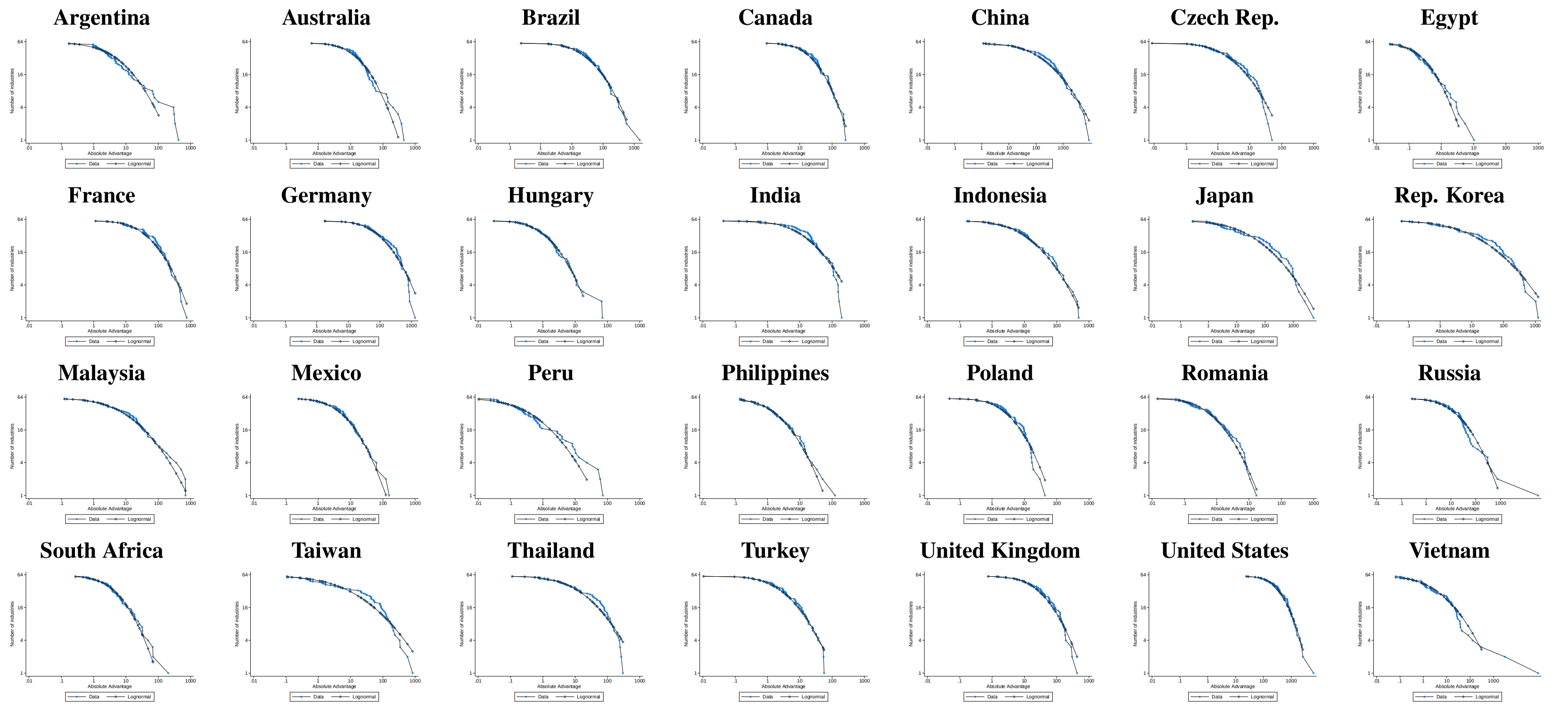
Source: WTF (Feenstra et al. 2005, updated through 2008) for 133 time-consistent industries in 90 countries from 2005-2007, three-year.
Note: The graphs show the frequency of industries (the cumulative probability $1 - F_{\hat{X}}(\hat{x})$ times the total number of industries $I = 133$) on the vertical axis plotted against the Balassa index of revealed comparative advantage $\hat{X} = (X_{is}/\sum_{\varsigma} X_{i\varsigma})/(\sum_j X_{js}/\sum_j \sum_{\varsigma} X_{j\varsigma})$ on the horizontal axis. Both axes have a log scale. The fitted log normal distribution is based on maximum likelihood estimation by country s in year $t = 2007$.

Figure S6: **Cumulative Probability Distribution of Absolute Advantage for 28 Countries in 1987, Two-digit Industries**



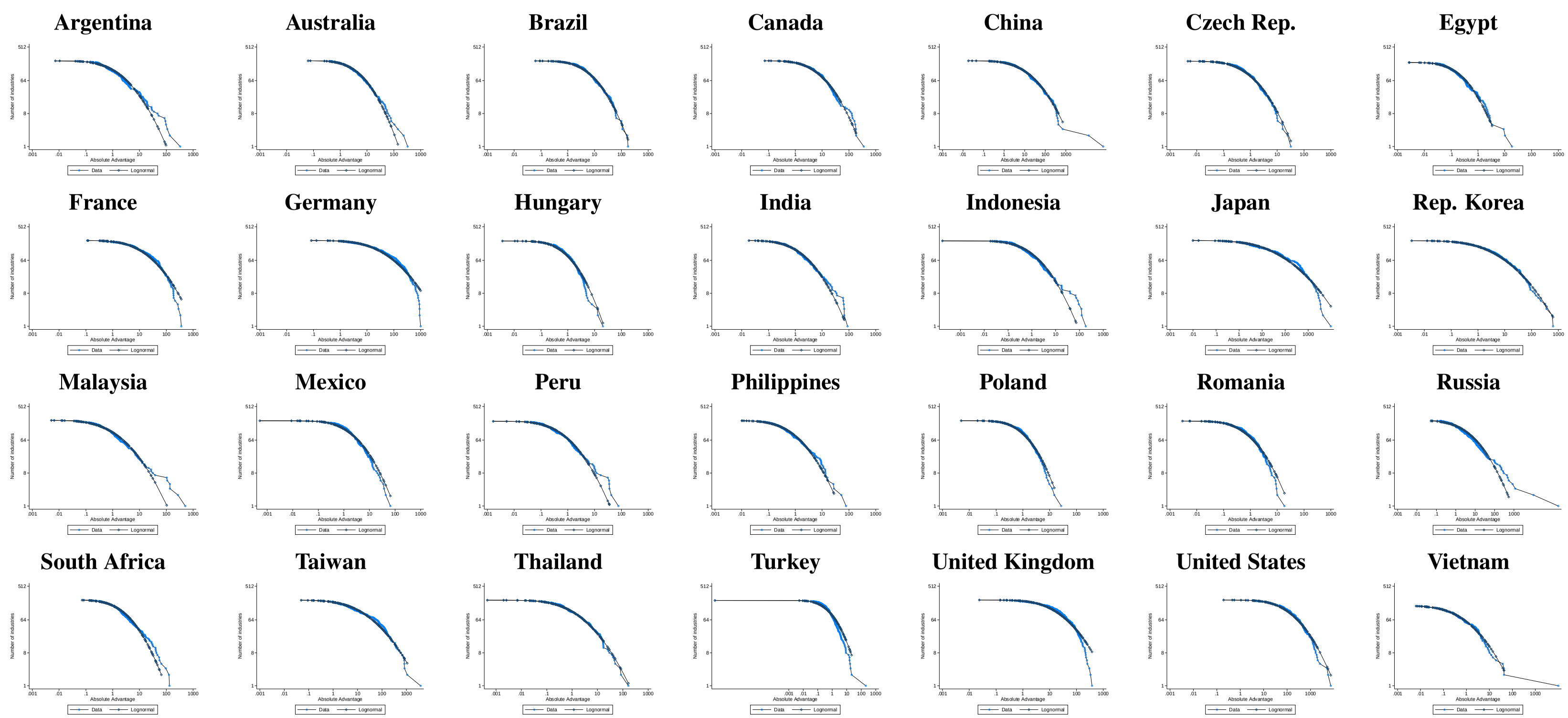
Source: WTF (Feenstra et al. 2005, updated through 2008) for 60 time-consistent industries at the two-digit SITC revision 2 level in 90 countries from 1985-1987 and CEPII.org; three-year means of OLS gravity measures of export capability (log absolute advantage) $k = \ln A$ from (6).
Note: The graphs show the frequency of industries (the cumulative probability $1 - F_A(a)$ times the total number of industries $I = 683$) on the vertical axis plotted against the level of absolute advantage a (such that $A_{ist} \geq a$) on the horizontal axis. Both axes have a log scale. The fitted log normal distribution are is based on maximum likelihood estimation by country s in year $t = 1987$.

Figure S7: **Cumulative Probability Distribution of Absolute Advantage for 28 Countries in 2007, Two-digit Industries**



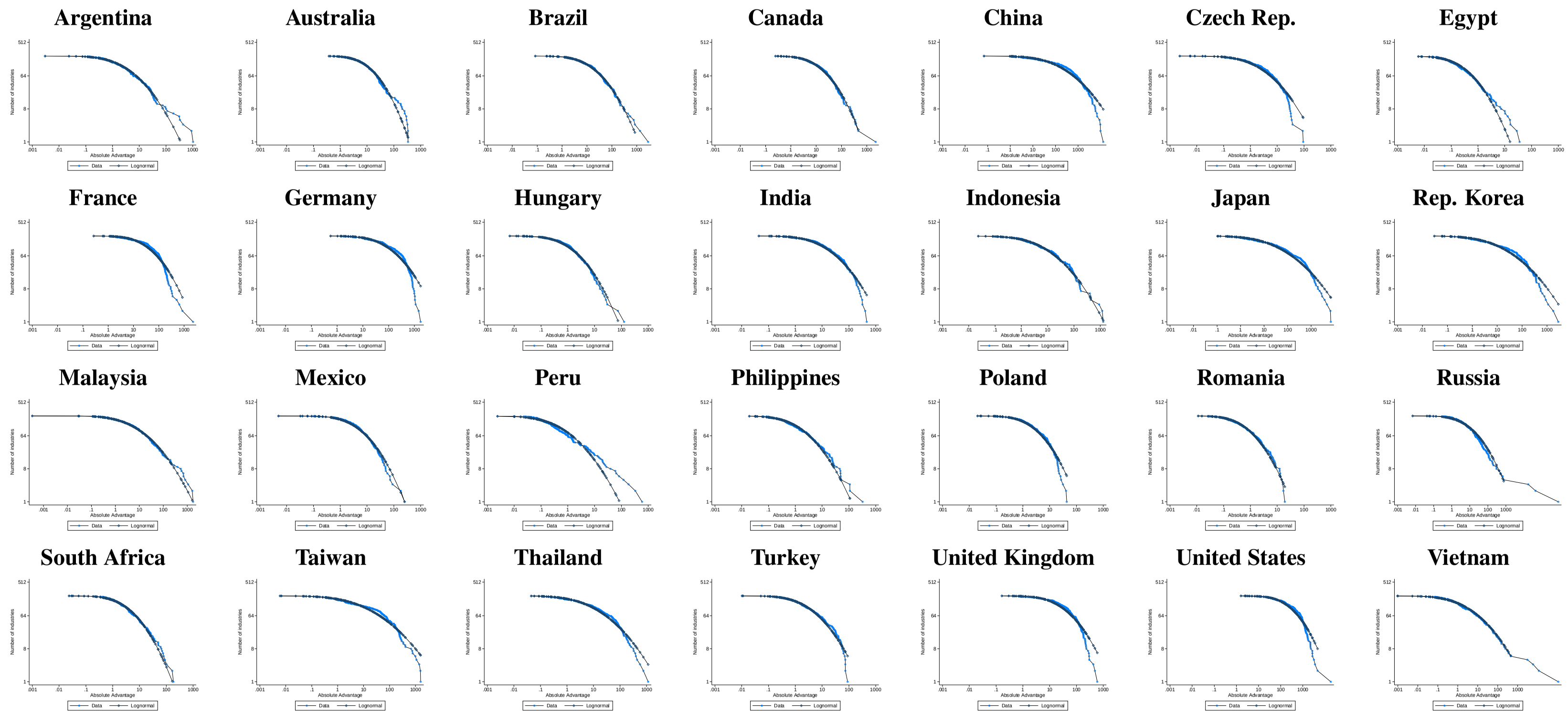
Source: WTF (Feenstra et al. 2005, updated through 2008) for 60 time-consistent industries at the two-digit SITC revision 2 level in 90 countries from 2005-2007 and CEPII.org; three-year means of OLS gravity measures of export capability (log absolute advantage) $k = \ln A$ from (6).
Note: The graphs show the frequency of industries (the cumulative probability $1 - F_A(a)$ times the total number of industries $I = 683$) on the vertical axis plotted against the level of absolute advantage a (such that $A_{ist} \geq a$) on the horizontal axis. Both axes have a log scale. The fitted log normal distribution are is based on maximum likelihood estimation by country s in year $t = 2007$.

Figure S8: **Cumulative Probability Distribution of Absolute Advantage for 28 Countries in 1987, Three-digit Industries**



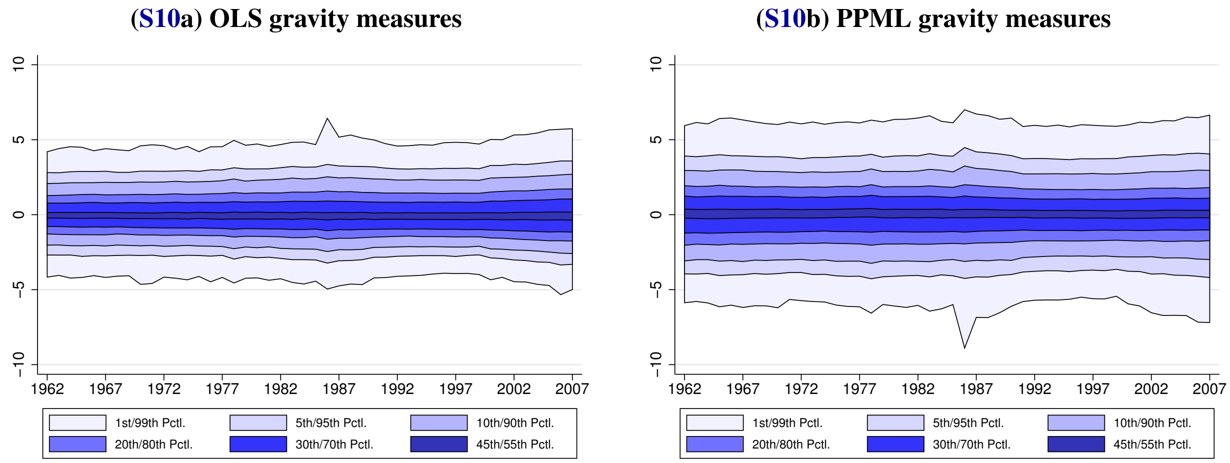
Source: WTF (Feenstra et al. 2005, updated through 2008) for 224 time-consistent industries at the three-digit SITC revision 2 level in 90 countries from 1985-1987 and CEPII.org; three-year means of OLS gravity measures of export capability (log absolute advantage) $k = \ln A$ from (6).
Note: The graphs show the frequency of industries (the cumulative probability $1 - F_A(a)$ times the total number of industries $I = 683$) on the vertical axis plotted against the level of absolute advantage a (such that $A_{ist} \geq a$) on the horizontal axis. Both axes have a log scale. The fitted log normal distribution are is based on maximum likelihood estimation by country s in year $t = 1987$.

Figure S9: **Cumulative Probability Distribution of Absolute Advantage for 28 Countries in 2007, Three-digit Industries**



Source: WTF (Feenstra et al. 2005, updated through 2008) for 224 time-consistent industries at the three-digit SITC revision 2 level in 90 countries from 2005-2007 and CEPII.org; three-year means of OLS gravity measures of export capability (log absolute advantage) $k = \ln A$ from (6).
Note: The graphs show the frequency of industries (the cumulative probability $1 - F_A(a)$ times the total number of industries $I = 683$) on the vertical axis plotted against the level of absolute advantage a (such that $A_{ist} \geq a$) on the horizontal axis. Both axes have a log scale. The fitted log normal distribution are is based on maximum likelihood estimation by country s in year $t = 2007$.

Figure S10: **Percentiles of Comparative Advantage Distributions by Year, OLS- and PPML-based Measures**



Source: WTF (Feenstra et al. 2005, updated through 2008) for 133 time-consistent industries in 90 countries from 1962-2007; OLS and PPML gravity measures of export capability (log absolute advantage) $k = \ln A$ from (6) and (8).
Note: We obtain log comparative advantage as the residuals from OLS projections on industry-year and source country-year effects (δ_{it} and δ_{st}) for (a) OLS and (b) PPML gravity measures of log absolute advantage $\ln A_{ist}$. Panel (a) repeats Panel (a) of Figure 3

S.7 GMM estimates of comparative advantage diffusion at ten-year horizon

We repeat GMM estimation of the generalized logistic diffusion of comparative advantage at the ten-year horizon. **Table S3** shows that estimated coefficients at the ten-year horizon are comparable to those for our benchmark estimation at the five-year horizon. The qualitative similarity in global diffusion coefficients at varying intervals for the estimation moments suggest that our results tightly characterize the dynamics of comparative advantage.

Table S3: GMM Estimates of Comparative Advantage Diffusion, 10-year Transitions

	OLS gravity k			PPML gravity k			$\ln RCA$		
		LDC	Nonmanf.		LDC	Nonmanf.		LDC	Nonmanf.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Estimated Generalized Logistic Diffusion Parameters									
Dissipation rate η	0.264	0.280	0.269	0.188	0.171	0.156	0.225	0.202	0.196
	(0.004)***	(0.006)***	(0.005)***	(0.048)***	(0.006)***	(0.166)	(0.012)***	(0.013)***	(0.009)***
Intensity of innovations σ	0.569	0.648	0.659	0.661	0.740	0.736	0.604	0.672	0.618
	(0.007)***	(0.012)***	(0.012)***	(0.263)**	(0.046)***	(0.786)	(0.076)***	(0.085)***	(0.042)***
Elasticity of decay ϕ	-0.029	-0.055	-0.026	0.013	0.024	-0.030	0.026	0.016	0.003
	(0.014)**	(0.025)**	(0.014)*	(0.276)	(0.043)	(0.685)	(0.092)	(0.095)	(0.041)
Implied Parameters									
Log gen. gamma scale $\ln\hat{\theta}$	202.43	82.33	232.38	-523.43	-239.70	173.77	-217.95	-404.48	-2,856.50
	(131.469)	(53.268)	(166.775)	(13961.730)	(591.202)	(5513.348)	(1017.349)	(3060.931)	(41377.530)
Log gen. gamma shape $\ln\kappa$	5.781	4.527	6.002	6.967	5.710	5.171	5.770	6.626	9.733
	(0.968)***	(0.897)***	(1.077)***	(41.667)	(3.681)	(44.979)	(6.974)	(11.679)	(24.111)
Mean/median ratio	7.281	7.119	6.932	13.329	16.019	32.282	8.392	10.969	12.627
Observations	335,820	211,640	161,940	332,320	209,760	158,750	335,820	211,640	161,940
Industry-source obs. $I \times S$	11,213	7,556	5,588	11,203	7,546	5,581	11,214	7,556	5,589
Root mean sq. forecast error	1.876	2.039	1.979	2.023	2.166	2.243	1.887	2.049	2.003
Min. GMM obj. ($\times$ 1,000)	3.03e-12	1.20e-11	1.53e-11	1.59e-11	4.83e-11	4.83e-11	2.01e-11	6.24e-11	8.82e-11

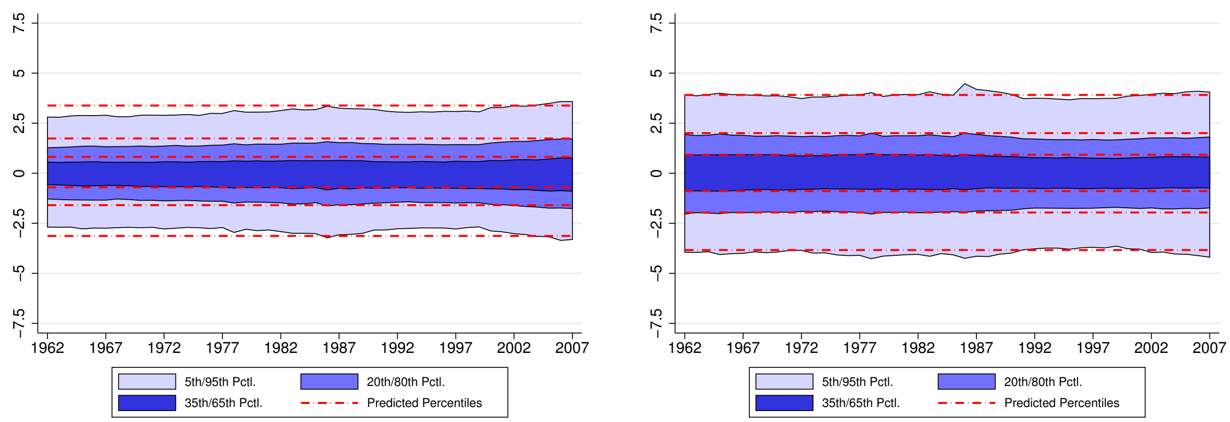
Source: WTF (Feenstra et al. 2005, updated through 2008) for 133 time-consistent industries in 90 countries from 1962-2007 and CEPII.org; OLS and PPML gravity measures of export capability (log absolute advantage) $k = \ln A$ from (6) and (8).
Note: GMM estimation at the ten-year horizon for the generalized logistic diffusion of comparative advantage $\hat{A}_{is}(t)$,

$$\mathrm{d}\ln\hat{A}_{is}(t) = -\frac{\eta\sigma^2}{2}\frac{\hat{A}_{is}(t)^{\phi}-1}{\phi}\,\mathrm{d}t + \sigma\,\mathrm{d}W^{\hat{A}}_{is}(t)$$

using absolute advantage $A_{is}(t) = \hat{A}_{is}(t)Z_s(t)$ based on OLS and PPML gravity measures of export capability k from (6), and the Balassa index of revealed comparative advantage $RCA_{ist} = (X_{ist}/\sum_{\varsigma} X_{i\varsigma t})/(\sum_j X_{jst}/\sum_j \sum_{\varsigma} X_{j\varsigma t})$. Parameters η, σ, ϕ are estimated under the constraints $\ln\eta, \ln\sigma^2 > -\infty$ for the mirror Pearson (1895) diffusion of (20), while concentrating out country-specific trends $Z_s(t)$. The implied parameters are inferred as $\hat{\theta} = (\phi^2/\eta)^{1/\phi}$, $\kappa = 1/\hat{\theta}^{\phi}$and the mean/median ratio is given by (A.10). Less developed countries (LDC) as listed in the Supplementary Material (Section S.1). The manufacturing sector spans SITC one-digit codes 5-8, the nonmanufacturing merchandise sector codes 0-4. Robust errors in parentheses (corrected for generated-regressor variation of export capability k): * marks significance at ten, ** at five, and *** at one-percent level. Standard errors of transformed and implied parameters are computed using the multivariate delta method.

Figure S11: **Fit to Percentiles of Comparative Advantage Distributions by Year, OLS- and PPML-based Measures**

(S11a) OLS gravity measures **(S11b) PPML gravity measures**



Source: WTF (Feenstra et al. 2005, updated through 2008) for 133 time-consistent industries in 90 countries from 1962-2007; OLS and PPML gravity measures of export capability (log absolute advantage) $k = \ln A$ from (6) and (8).
Note: The graphs depict the observed percentiles as previously shown in Figure S10 and the predicted percentiles from the GMM estimates of the comparative advantage diffusion (17) in Table 2 (parameters η and ϕ in columns 1 and 4). Observed log comparative advantage is based on the residuals from OLS projections on industry-year and source country-year effects (δ_{it} and δ_{st}), absorbing the country-specific stochastic trend component $\ln Z_{st}$ from (18), for (a) OLS and (b) PPML gravity measures of log absolute advantage $\ln A_{ist}$.