Abstract: Competition between microfinance institutions (MFIs) in developing countries has increased dramatically in the last decade. We model the behavior of non-profit lenders, and show that their non-standard objectives cause them to cross-subsidize within their pool of borrowers. As a result, when lenders with heterogeneous objectives compete, competition is likely to yield an outcome that makes poor borrowers worse off. As competition exacerbates asymmetric information problems over borrower indebtedness, the most impatient borrowers begin to obtain multiple loans, creating a negative externality that leads to less favorable equilibrium loan contracts for all borrowers.

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1. Introduction

Since Adam Smith, economists have nearly always favored policies that foster competition, as competition typically results in lower equilibrium prices for consumers. It would be reasonable to expect therefore that an increase in competition between microfinance institutions would unequivocally result in more favorable credit contracts for the entrepreneurial poor in developing countries. The purpose of this paper, however, is to show that competition may actually prove detrimental to some or all of the borrowers in a microfinance market.

We develop a model in which a solitary client-maximizing microfinance institution (MFI) competes with an existing informal moneylender to the benefit of each borrower captured in the microfinance portfolio. Subsequently we show three potentially adverse effects of the entrance of new MFIs into the same pool of borrowers. First, Bertrand competition between MFIs within the subset of profitable borrowers reduces the ability of a socially motivated lender to generate rents that support lending to the poorest and potentially least profitable borrowers. This diminution of the capacity to cross-subsidize means that the poorest borrowers in the client-maximizing portfolio are dropped as competition intensifies. Second, we show a number of instances in which failure to restrict grant funding to the poorest potential borrowers can prevent the emergence of a competitive microfinance market altogether, as client-maximizing non-profit institutions undercut profit-maximizers to capture the most profitable borrowers in a given pool.

A third negative effect of MFI competition originates from the likelihood of increasing asymmetric information between lenders. With a greater number of lenders in a market, we would expect information sharing between lenders to become more difficult, all else equal. We show that this creates an incentive for some (impatient) borrowers to take multiple loans. Such instances of multiple contracting both increase average debt levels among borrowers in the portfolio and decrease the expected equilibrium repayment rate on all loan transactions, generating less-favorable Bertrand

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1 See Shaffer (1996) for an example of cross-subsidization in U.S. lending markets.
equilibrium credit contracts. This makes all patient borrowers worse off, and again results in the poorest borrowers being dropped from the loan portfolio. In general, our results show that while wealthier and impatient borrowers are likely to benefit from increasing competition among MFIs, very plausible conditions exist under which an increase in the number of lenders in a market will lower the welfare of the both the poor and the patient.

The widespread enthusiasm for microfinance has spawned a dramatic increase in the number of MFIs in the developing world. Spurred by an accord reached at the Microfinance Summit in 1997 to reach 100 million of the world's poorest households with credit, there is arguably more widespread support for microfinance today than any other single tool for fighting world poverty. The microfinance movement has been both praised and supported by a broad range of academic scholars, major development finance institutions such as the World Bank, and development practitioners themselves. With the number of MFIs involved in this effort now 1,600 and growing, the overlap and competition between these institutions is certain to increase.

The rapid early growth of the microfinance movement primarily consisted of non-profit, socially motivated lenders seeking to reach as many poor clients with credit as they were able, given their limited budgets. In the process they demonstrated that through the use of new lending technologies, such as joint liability contracts and dynamic incentives, a substantial portion of this new market could in fact be lent to profitably. This realization has drawn profit-motivated lending institutions into these markets. The presence of competition from profit-driven lenders has forced MFIs in competitive regions to rethink their strategies. Moreover, donors have questioned the need for continued subsidies, resulting in the recent focus on “institutional sustainability” in the MFI sector.

Although some current research has begun to touch upon these issues, most of the burgeoning economic literature on MFIs has been concerned with the impact on MFI clients, (e.g. Pitt and Khandker, 1998; Morduch, 1998; Wydick, 1999) or papers that examine the properties of the joint

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2 Tuckman (1998) provides a good survey of competition among non-profit institutions.
liability contracts often employed in microenterprise lending (e.g. Stiglitz, 1990; Besley and Coate, 1995; Ghatak, 1999). A limited amount of work, however, has undertaken a broader look at the industrial organization of the microfinance movement.

Morduch’s (1999, 2000) work is one such example. In a detailed analysis of the Grameen Bank, Morduch (1999) asserts that the failure to account for tradeoffs between sustainability and poverty reduction has hamstrung discussion about the subsidies necessary for microfinance to move forward. In a later paper (Morduch, 2000), he challenges the notion that microfinance provides a ‘win-win’ situation for all players involved. Instead, he argues that subsidized lending to the poor as well as the creation of sustainable for-profit institutions should be important, but separate, development goals.

Navajas, Conning, and Gonzales-Vega (2001) apply a variant of Conning’s (1999) model to lenders in Bolivia, showing that a high-screening, individually focused institution and a low-screening group-lender with poorer clients can coexist in the same market. While their research discusses the client sorting process that will take place under competition, its focus is different from our work in that it does not explicitly model interactions between lenders in terms of strategic behavior and multiple contracting by borrowers when more than one lender exists in the market. Rather, they model the second entrant to the market as a Stackelberg follower who focuses on collateral-based lending and hence captures the wealthiest and most productive segment of the market.

More similar to this paper in its approach to strategic behavior is Hoff and Stiglitz (1998), who show that subsidies to a lending market can generate the perverse consequences of market exit and increasing equilibrium prices. In their model, these effects can arise either as a result of the loss of scale economies, or due to a weakening of reputation effects as the number of lenders dilutes information in the market. We also relate information asymmetries to poorer equilibrium loan contracts for borrowers in Section 4 of this paper. However, in the Hoff and Stiglitz model, perverse effects are caused by the weakening of dynamic repayment incentives, whereas in our model it is caused by asymmetric information between lenders over borrower indebtedness.
Other papers have also examined the industrial organization of credit markets. Petersen and Rajan (1995) show that multi-period, state-contingent contracts, or “relationship banking” is an efficient contracting device for dealing with asymmetric information, and for this reason market power can lead to lower quality firms obtaining finance. Similarly, since the screening required in multi-lender markets is wasteful, the authors demonstrate that borrower welfare can decrease as competition intensifies. Broecker (1990), examining an equilibrium under Bertrand competition, shows that imperfect information provides an additional reason for undercutting, as offering the prevailing interest rate causes the lender to attract only those borrowers rejected by other lenders. Dell’Ariccia et al. (1999), illustrate a “blockaded” market, where Bertrand competition leads to no more than two lenders being present in a market in equilibrium. Marquez (2002), however, presents a model where entry is easier as turnover increases, weakening blockading. A more applied view of the escalating competition present in microfinance markets is provided by Rhyne and Christen (1999), who suggest that information sharing in the form of credit bureaus is becoming increasingly necessary as the microfinance market matures.

2. Increasing Numbers and Competition among MFIs

We wish to motivate our theoretical model with evidence from three areas of the world in which MFI activity has reached a relatively advanced stage, and where the effects of competition between MFIs have become increasingly clear.

**Bangladesh:** The Grameen Bank, long the flagship of the microfinance movement, has consistently been upheld as a pinnacle of stability, self-sufficiency, and effectiveness in using microfinance as a tool for lifting households from poverty. Yet the Grameen Bank's well-known successes have encouraged imitators, which compete for borrowers’ attention along with two other very large microcredit providers, Bangladesh Rural Advancement Committee (BRAC) and Rural Development Project 12 (RD-12), that have operated alongside the Grameen Bank for more than a decade. A front page *Wall Street Journal* article in November 2001 raised warnings about the financial
health of the Grameen Bank, asserting “imitators have brought on more competition, making it harder
for Grameen to control their borrowers”. The article points in particular to the Grameen Bank’s lending
in the region of Tangail, in which competitive pressures have reduced interest rates for some borrowers,
but where 32.1 percent of the Grameen Bank’s loans have fallen more than two years overdue:

In Tangail, signboards for rival Micro lenders dot a landscape of gravel roads, jute fields and ponds with
simple fishing nets. Shopkeepers playing cards in the village of Bagil Bazar can cite from memory the
terms being offered by seven competing microlenders—a typical repayment plan for a 1,000-taka ($17) loan
is 25 taka for 46 weeks. At an annualized rate, that works out to 30% in interest. Surveys have estimated
that 23% to 43% of families borrowing from microlenders in Tangail borrow from more than one.

(WSJ: 11/27/2001)

Such conditions in Bangladesh, with perhaps the world’s most developed microfinance industry, have
put huge financial pressures on lenders, and have led to spiraling default rates. Where once Grameen
boasted repayment rates of 95% or higher, the Wall Street Journal reports 19% of the portfolio overdue
by a year or more. Alarming figures such as these have intensified efforts by the World Bank and
CGAP to help bring together a network of the largest 20 microfinance institutions in Bangladesh to
implement a centrally managed credit information system during 2004. It is hoped that more “centrally
managed” competition between lenders in Bangladesh will both help to foster healthy competition
between MFIs while bringing down arrears rates in MFI portfolios.

East Africa: While East Africa is at an earlier stage of competition, the major urban centers of
Uganda and Kenya are becoming saturated by competition among numerous MFIs (see Kaffu &
Mutesasira, 2003). Markets for the more wealthy borrowers that were previously dominated by grant-
funded, socially motivated lenders are now being contested by private institutions. For example,
CERUDEB and CMF, two private lenders with access to subsidized external lines of credit, have begun
competing with existing MFIs for larger microcredit borrowers. In response, there is increasing
competitive pressure on socially motivated MFIs, whose interest rates may be more than 1% per month
higher than the new competition.

FAULU, one of the few major MFIs to operate in both Uganda and Kenya, is troubled by the
increasing presence of borrowers unknowingly receiving loans from multiple lenders. FAULU reports
that such behavior has become increasingly prevalent as the intensity of MFI activity increases. The Kenya office is able to employ a risk management network based on the country’s national ID system to detect clients within their own portfolio with multiple loans. Uganda, however, has no such national ID system, and so they are powerless to monitor the problem, even within their own institution.

Central America: The increase in both the size and number of MFIs operating in Central America since the mid-1990s has been astounding. Much of the reason for this has been political: both the United States and the European Union have desired to develop an entrepreneurial middle-class in the region in order to try to bridge the societal divisions responsible for civil wars during the 1980s. As a result, growth in MFI activity has been particularly heavy in Guatemala, El Salvador, and Nicaragua.

Table 1: Growth in Microenterprise lending, Nicaragua

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<tr>
<td>ASODENIC</td>
<td>10,568</td>
<td>1,711,801</td>
<td>8.00</td>
<td>24,000</td>
<td>2,995,942</td>
<td>10.39</td>
</tr>
<tr>
<td>FAMA</td>
<td>6,087</td>
<td>6,230,864</td>
<td>0.21</td>
<td>16,402</td>
<td>6,948,907</td>
<td>1.63</td>
</tr>
<tr>
<td>FINCA</td>
<td>15,275</td>
<td>1,688,417</td>
<td>0.00</td>
<td>24,888</td>
<td>2,680,631</td>
<td>9.00</td>
</tr>
<tr>
<td>FUNDENUSE</td>
<td>2,129</td>
<td>913,945</td>
<td>4.40</td>
<td>5,940</td>
<td>1,609,631</td>
<td>5.20</td>
</tr>
<tr>
<td>ProMujer</td>
<td>10,121</td>
<td>317,870</td>
<td>1.25</td>
<td>10,561</td>
<td>629,385</td>
<td>0.35</td>
</tr>
<tr>
<td>Five-MFI Total</td>
<td>44,180</td>
<td>10,862,897</td>
<td>1.79</td>
<td>81,791</td>
<td>14,864,496</td>
<td>5.06</td>
</tr>
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Source: USAID Microfinance Results Reporting. Averages for 5-MFI Total are portfolio weighted.

The case of Nicaragua is typical of the region. FAMA, an ACCION International affiliate, enjoyed a virtual monopoly in microfinance lending in the Managua area for the few years after it commenced operations in 1992. However, by 1996 approximately six other major MFIs entered the market, though even by 1997 no MFIs in the region had a portfolio of more than 4,000 borrowers. Moreover, according to ASOMIF, the association of Nicaraguan microfinance institutions, the portfolios of Nicaraguan MFIs grew at an annual rate of 47% between 1997 and 2001 (La Prensa, 10/02/2002). By 2001 the largest MFIs were carrying portfolios in the range of 15,000-25,000 borrowers, with considerable overlap in geographical operating regions. Table 1 gives USAID data illustrating microlending growth during 1999-2001 of the five largest MFIs receiving USAID funding. Note that as portfolios have increased dramatically, levels of arrears have also risen in all but one case.
Guatemala and El Salvador have experienced similarly dramatic growth in MFI activity. FUNDAP in Quetzaltenango, Guatemala, like its sister ACCION institution in Nicaragua, experienced very little competition from other MFIs since its inception in 1988 until the mid-1990s. New entrants into its regional market, such as FUNDESPE and Fe y Alegría, have forced FUNDAP to cut interest rates on its larger loans from 3% to 2.5% per month. To remain solvent under competition, it has pulled away from its initial mission of offering smaller loans in the form of group-based credit, and instead now lends to a wealthier, more lucrative segment of the market. While average initial loan size was US$135 and average monthly sales were US$291 for borrowers receiving their first loans between 1988 and 1993, by 1999 these figures for new clients had grown to US$543 and US$672, respectively.3

Asymmetric information between lenders over borrower quality and indebtedness has been a mounting issue in all three countries, but there have been great differences between the three countries in the level of cooperation realized between MFIs to mitigate the problem. El Salvador, with its internet-driven Info-Red borrower database4, represents the best example of a case where a network of independent MFIs have built information-based institutions reminiscent of those in developed countries, where nearly instantaneous credit checks are possible. In Guatemala, multiple contracting by MFI clients had become so damaging by the late 1990s that REDIMIF, an association of 19 MFIs embarked on an effort to establish CREDIREF, a centralized microfinance credit bureau, which though now functional, is still in its nascent stages. Cooperation between MFIs during the mid-1990s was fairly strong in Nicaragua; institutions regularly shared information on poorly performing borrowers with one another. However, as MFIs poured into the market in the late 1990s, cooperation has deteriorated to such an extent that, as one loan officer put it "our information-sharing consists of a trip to the local cantina to ask neighbors if loan officers from other MFIs have been paying visits to a potential client.”

3 Figures are inflation-adjusted. Source: sample of 376 field surveys conducted in Guatemala in 1994-1999.
3. Basic Model: Full Information

3.1 Analytical framework

Consider an MFI operating in a large pool of potential borrowers who, for well-established reasons, are denied access to credit in the formal financial sector. New lending technologies such as group lending, community banking, and dynamic incentives (as well as the possibility of grant funding) have allowed an MFI access to these borrowers. Let this pool of potential borrowers be defined by the set \( \mathcal{B} \equiv \{1, 2, \ldots, n\} \) where \( \mathcal{B} \) is indexed in ascending order of an initial level of productive assets \( k_i \), pertaining to each \( i \in \mathcal{B} \). The corresponding set of initial assets \( \mathcal{K} \equiv \{k_1, k_2, \ldots, k_n\} \) is observable to the MFI and uniformly distributed within \( \mathcal{B} \) such that \( k_i \) is the lowest level of endowment, \( k_1 \), and the highest, \( k_n \), as \( \bar{k} \).

For each borrower \( i \in \mathcal{B} \), the MFI considers granting a loan \( V_i \) at an interest rate \( r_i \). The interest cost of capital for the MFI is equal to \( c \), and on each loan it incurs fixed administrative costs \( F \). The loan offered by the MFI is subject to a participation constraint, representing the best alternative source of financing by borrower \( i \). We will represent this best alternative financing as offered by a local moneylender competing over the same set of potential borrowers \( \mathcal{B} \), offering loans that return a profit \( \Pi_i(V^m_i, r^m_i) \) to borrower \( i \in \mathcal{B} \). The moneylender has lower fixed costs per loan than the MFI, \( F^m < F \), but a higher cost of capital, \( c^m > c \), and thus operates at a cost disadvantage (advantage) for loan sizes \( V_i > \left( \frac{F - F^m}{c^m - c} \right) \). We assume that prior to the entry of the MFI, the monopolistic moneylender operates in a predatory manner as in Basu (1984), such that the moneylender is able to induce any borrower \( i \) to accept a loan contract that leaves his client with only some very small level of profit, \( \varepsilon > 0 \).

If a borrower receives a loan, the resulting investment yields a low return per unit of borrowed capital of \( \beta < 1 \) with probability \( p_i(k_i, V_i) \) due to a lack of ability to post sufficient collateral, and a high
return of $\beta > 1 + r_i$ with probability $1 - p_i(k_i, V_i)$. We posit that the probability of the low return, in which the borrower is forced to default on $1 - \beta$ of the principal is decreasing in $k_i$ and increasing in $V_i$. This is equivalent to saying that a wealthier individual has a higher probability of repaying a loan of a given size than a poorer individual, and that increasing the size of the loan to a given individual cannot increase the probability of repayment.\(^5\) To simplify our model, we assume $p_v > 0$ and $p_k < 0$ to be constant and that $p_i = \beta + p_v V_i + p_k k_i$.\(^6\) We abstract from strategic default in the sense that any project yield up to and including $(1 + r_i)V_i$ is forfeited to the lender. Profit to the borrower is thus

$$
\Pi_i^B = (1 - p_i)(\beta - (1 + r_i))V_i
$$

while profit to the MFI is

$$
\Pi_i^{MFI} = (1 - p_i)(1 + r_i)V_i + p_i \beta V_i - (1 + c)V_i - F.
$$

The profit function of the moneylender is identical to that of the lending institution, except for the moneylender’s lower fixed costs per loan, $F^m$, and higher cost of capital, $c^m$. The slope of the borrower’s isoprofit curves can be derived through total differentiation of (1) to obtain

$$
\frac{dV_i}{dr_i} = \frac{-V_i(1 - p)}{(\beta - (1 + r_i))(p_v V_i - (1 - p))}. \tag{3}
$$

The isoprofit curves for the borrower are strictly decreasing in $r_i$ and backward-bending with respect to $V_i$ because the borrower fears default as the loan becomes too large. Similarly we totally differentiate (2) to obtain the slope of the lender’s isoprofit curves:

$$
\frac{dV_i}{dr_i} = \frac{V_i(1 - p_i)}{(p_v V_i + p_i)(1 + r_i - \beta) - (r_i - c)}.
$$

The lender’s isoprofit curves are increasing in $r_i$ and also backward-bending in $V_i$ due to the risk of default in large loans. The lender’s isoprofit curves are positively (negatively) sloped for values of

\(^5\) Boundary conditions on parameters given in the Appendix guarantee that $p_i \in (0,1)$ for all $k_i \in [k, \bar{k}]$.

\(^6\) For analytical tractability, we assume $p$ to be independent of the interest rate.
\[ V_i > (\frac{1}{2p_v}) \left( \frac{(r_i - c)}{(1 + r_i - \beta)} - \bar{p} - p_k i \right) \]  Note that \( c^m > c \) implies that the indifference curves of the moneylender are steeper on the negatively sloped portion and flatter on the positively sloped portion; the critical \( V_i \) at which the slope becomes vertical is also lower than for the MFI. The borrower’s isoprofit curves at \( r_i < \bar{\beta} - 1 \) are positively (negatively) sloped for \( V_i < (>) (1 - \bar{p} - p_k i) / 2p_v \). This implies that the value of \( V_i \) at which the borrower’s isoprofit curve bends backward is higher than that for the lender if \( 1 + c \) and \( 1 + c_m > \beta \) (see Assumption 3 below). This and other assumptions in our model include the following:

**Assumption 1**: “Moneylender Feasibility”. In the absence of competition, the moneylender can offer some loan to every borrower in the pool that is weakly profitable for both. In other words for each \( i \in B \), \( \exists \) some \((V_i, r_i)\) for which \( \Pi_{ML}(V(k_i), r(k_i)) \geq 0 \) and \( \Pi^B(V(k_i), r(k_i)) \geq 0 \). This implies \( k \) is larger than some minimum value whose terms, given in the parameters of the model, are provided in the Appendix.

**Assumption 2**: “Microfinance Lending Environment”. When an MFI competes with a moneylender, there exists more than one profitable borrower and at least one unprofitable borrower for the MFI in the borrower pool. In other words, at its optimal loan contract \((V_i^*, r_i^*)\) for each \( i \in B \), we have \( \sum_{i \in \Pi_{MFI}(V^*, r^*) > 0} i > 1 \) and \( \sum_{i \in \Pi_{MFI}(V^*, r^*) < 0} i \geq 1 \). The parameters that satisfy this condition are also detailed in the Appendix.

**Assumption 3**: “Potential Lender Loss”. We assume both the moneylender and the MFI will lose money in the bad state of nature for the borrower, or specifically that \( 1 + c_m \) and \( 1 + c > \beta \). This is consistent with what we expect to be true in a developing-country context: a lender will lose money under default and will require more interest to risk larger loans in equilibrium. It implies that equilibrium contracts reside on the positive slope of both lender and borrower isoprofit curves. These assumptions are necessary for equilibrium calibration of the model; we also believe they reflect the lending environment within which most MFIs operate in the developing areas of the world.
3.2 The MFI as client-maximizer

The behavior of a profit-maximizing lender is well understood, but less clear is the objective of a non-profit MFI, as are the great preponderance of NGOs involved in microlending. We argue here that the objective of such an institution is typically to maximize its “outreach”, or the number of clients, \( n^* \), captured into its portfolio. The constraints are a net budget-balancing requirement, the participation constraint, and non-negativity constraints. A non-profit, client-maximizing MFI which (potentially) receives grant funding, \( G \), therefore solves the following problem:

\[
\begin{align*}
\text{Maximize} & \quad n^* \\
\text{subject to} & \quad \begin{align*}
\text{BC:} & \quad \sum_i \Pi_i^{\text{MFI}}(V_i, r_i) + G \geq 0 \\
\text{PC:} & \quad \Pi_i^B(V_i, r_i) \geq \Pi_i^B(V_i^m, r_i^m) \\
\text{NC:} & \quad V_i, r_i \geq 0
\end{align*}
\end{align*}
\]

for all \( i \) captured in the MFI borrower portfolio.

The intuition of the MFI problem, given more formally in the proofs of LEMMAS 1 and 2, is straightforward, and is as follows: The MFI maximizes its number of clients, \( n^* \), by first capturing the set of profitable borrowers, \( \mathcal{B}^p \), from the moneylender. This is accomplished though offering a contract\(^7\) \((V_i, r_i)\) to \( i \in \mathcal{B}^p \) that leaves each \( i \in \mathcal{B}^p \) indifferent to the best contract the moneylender is able to offer (at which \( \Pi_i^{\text{ML}}(V_i^m, r_i^m) = 0 \) and the PC binds). These equilibrium contracts are profitable for the MFI but not to the moneylender, since on larger loans, the MFI’s lower cost of capital more than makes up for its higher fixed lending costs. To maximize the number of borrowers in its portfolio, the surplus, \( A \), from lending to \( \mathcal{B}^p \) is used to subsidize loans to \( \mathcal{B}^{\text{NP}} \), a subset of the non-profitable borrowers \( \mathcal{B}^{\text{NP}} \).

\(^7\) Though little variation in interest rates is often observed in less competitive microfinance markets, much greater variation in interest rates among different types of borrowers is observed as financial markets mature and become more competitive, as seen for example in economies with highly developed financial systems. In the more mature microfinance markets in Central America, we are already beginning to observe competitive pressure giving birth to multiple loan products at differing interest rates that are targeted at a borrowers with varying levels of initial assets.
\( \tilde{B}_i^{NP} \) thus consists of the set of \( m^* \) unprofitable borrowers captured into the portfolio by means of cross subsidy A and grant funding G. Cross-subsidization begins with the borrowers \( \{i_k - 1, i_k - 2, \ldots \} \) on whom losses are the smallest on the MFI contract at the point where the PC binds. The number of unprofitable borrowers \( m^* \) in the portfolio is then maximized by offering contracts to increasingly poorer borrowers until the BC binds. Borrowers too poor to be reached by the MFI receive the exploitative monopoly moneylender contract for which \( \Pi_i^B(V_i^m, r_i^m) = \epsilon \). \(^8\) The following lemmas are foundational for the propositions that follow:

**Lemma 1:** There exists a Pareto-efficient contract curve for every loan made between a lender and each borrower \( i \).

**Proof:** See the Appendix.

By setting (3) equal to (4) we obtain the tangency points between the isoprofit curves of the MFI and borrower \( i \). This condition reduces to

\[
(p, V + p_i)\left(\bar{\beta} - \beta\right) - \left(\bar{\beta} - (1 + c)\right) = 0
\]

which forms an Edgeworth-like contract curve for each borrower given by the dark horizontal curves in Figures 1a and 1b. (The moneylender’s contract curve is shown by the lower dashed line.) By substituting the default function into (5a) we obtain

\[
V_i^*(k_i) = \frac{1}{2p_i} \left[ \bar{\beta} - (1 + c) \right] - \bar{p} - p_k k_i
\]

noting that the moneylender’s optimal contract is identical except that \( V_i^{m*}(k_i) < V_i^*(k_i) \) since \( c^m > c \).

The contract curves are linear and horizontal, meaning that differences in the contractual interest rate represent a direct transfer of profit between the borrower and lender. Figure 1a gives the example of a borrower \( i \in \tilde{B}^P \), for whom the MFI surplus is positive where the PC is binding. In Figure 1b, the

\(^8\) If we relax Assumption 1, then the MFI may able to reach poorer borrowers than the moneylender. This complicates the model, though the main results of the model are unchanged except that in order to reach the maximum number of borrowers the true client-maximizing MFI will extract the entire surplus from loans to such borrowers.
surplus is negative to the MFI for a borrower \( i \in \bar{B}^{NP} \) at the point where the PC binds. It is important to see how borrower welfare increases by virtue of entry from the MFI, moving from points \( A \) to \( A^* \) and \( B \) to \( B^* \), respectively. Entry of the MFI makes borrower profits rise and moneylender profits fall, so that the equilibrium contract changes from a contract that lies at zero profit for the borrower to a contract that lies at zero profit for the moneylender for all \( i \in \mathcal{B}^p \cup \bar{B}^{NP} \).

** Lemma 2:** The client-maximizing objective yields an equilibrium loan contract in which (i) \( V^*(k_i) \) is increasing in \( k_i \), (ii) equilibrium profit to the MFI is increasing in \( k_i \), (iii) lending to poorer borrowers is subsidized by profits from wealthier borrowers, (iv) a unique number of clients receive loans from the MFI for any microfinance subsidy \( G \geq 0 \).

** Proof:** See the Appendix.

In Figure 2 we can observe the two pivotal asset endowments: \( \tilde{k} \in \mathcal{K} \) is the asset endowment of the break-even borrower, while \( \tilde{k} \in \mathcal{K} \) is the asset endowment of the borrower \( i \in \bar{B}^{NP} \) who causes the BC to bind, and so is the poorest agent offered credit by the client-maximizing MFI. We denote \( \tilde{k} \) as the poorest agent served by the MFI in the absence of grant funding and \( \tilde{k} \) as the poorest served in the presence of some \( G > 0 \).

We believe the equilibrium illustrated in **Lemma 2** is the typical case of microfinance markets.

In a borrower pool in which all borrowers have extremely low initial endowments and \( \mathcal{B}^p = \{\emptyset\} \), only grant-funded agencies can survive. In cases in which all borrowers have very high endowments and
\[ B^{NP} = \{ \emptyset \} \], we should observe all lending conducted by for-profit lenders rather than grant-supported NGOs. The typical microfinance market is the intermediate case where cross-subsidization within a given client pool is possible, and the losses realized on loans to borrowers \( \in B^{NP} \) can be covered by a combination of grants and profits made from lending to borrowers \( \in B^P \). One implication of this is that when there is a single client-maximizing MFI that has access to the borrowing pool, there is no need to target grant funding towards the poor in this intermediate case. All surplus is automatically directed to the poorest clients. Figure 2 illustrates the surplus by borrower across the set \( \mathcal{K} \):  

\[ \Sigma \Pi_i^{MFI}, \text{ subject to:} \]

\[ PC: \Pi_i^B (V^+, r^+), \Pi_i^B (V^m, r^m) \] (Participation Constraint)

\[ 9 \] Two examples in Guatemala are BanRural and BanCafe, formal lenders that have entered the microfinance market, and now are major providers of microcredit in the country; another example is the well-known BancoSol in Bolivia.
Based on their respective objective functions, Bertrand competition by these identical MFIs will take place over \( i \in \mathcal{B}^p \), which induces undercutting for all borrowers for whom the PC and the NC are satisfied. Under competition solely with the moneylender, the PC of the cross-subsidizing MFI’s binds for all borrowers \( i \in \mathcal{B}^p \cup \widetilde{\mathcal{B}}^\text{NP} \) at the level where \( \Pi_i^{ML}(V_i^m, r_i^m) = 0 \). For the profit-maximizing MFI under competition, it is the NC that binds for all \( i \in \mathcal{B}^p \) because the incumbent MFI, using identical technology, then defines the PC. For the break-even borrower \( i \in \mathcal{B} \), both constraints are binding under both market structures, and so the contract offers are the same. We denote the equilibrium MFI contract under moneylender competition as \( (\hat{V}_i^*, r_i^*) \), and the Bertrand competitive equilibrium after entry of a second MFI by \( (\hat{V}_i^{**}, r_i^{**}) \).

**Proposition 1:** Bertrand competition between MFIs benefits wealthier borrowers, but makes a group of poorer borrowers worse off.

**Proof:** The proof follows in a straightforward manner from Lemmas 1 and 2. Under Bertrand competition between MFIs, \( \Pi_i^{MF}(V_i^{**}, r_i^{**}) \to 0 \) for \( \forall i \in \mathcal{B}^p \). Since by Lemma 1 all contracts in equilibrium are Pareto efficient, this implies that \( \Pi_i^B(\hat{V}_i^{**}, r_i^{**}) > \Pi_i^B(V_i^*, r_i^*) \) for \( \forall i \in \mathcal{B}^p \). Because \( \Pi_i^{MF}(V_i^{**}, r_i^{**}) = 0 \) for \( \forall i \in \mathcal{B}^p \) in equilibrium, this renders cross-subsidization impossible. Therefore, borrower profits for \( \forall i \in \widetilde{\mathcal{B}}^\text{NP} \) fall from \( \Pi_i^B(V_i^*, r_i^*) \) to \( \Pi_i^B(V_i^m, r_i^m) \) where \( (V_i^m, r_i^m) \) is the monopolistic moneylender contract. We know that \( \Pi_i^B(V_i^*, r_i^*) > \Pi_i^B(V_i^m, r_i^m) \) because when two MFIs compete in Bertrand equilibrium we have \( \Pi_i^{ML}(V_i^m, r_i^m) > 0 \) and \( \Pi_i^B(V_i^m, r_i^m) = \varepsilon \) for \( \forall i \in \widetilde{\mathcal{B}}^\text{NP} \). Conversely, with only a single MFI competitor the PC binds for \( \forall i \in \widetilde{\mathcal{B}}^\text{NP} \) by Lemma 2, implying that at \( \Pi_i^B(V_i^*, r_i^*) \) we have \( \Pi_i^{ML}(V_i^m, r_i^m) = 0 \). Since all contracts in equilibrium are Pareto efficient by Lemma 1, an equilibrium contract such that \( \Pi_i^{ML}(V_i^m, r_i^m) = 0 \) therefore implies that \( \Pi_i^B(V_i^*, r_i^*) > \varepsilon \). \( \square \)
Notice that faced with entry by a profit-maximizing MFI, the client-maximizing MFI responds by conforming to the behavior of a profit-maximizing MFI. Profits to the Bertrand competitors go to zero in equilibrium; all surplus is captured in higher profits to the wealthiest borrowers in $B$.

3.4 Competitive equilibrium with non-targeted subsidies

We will refer to a subsidy that has been earmarked by the donor for underwriting the costs of loans specifically to unprofitable (poorer) borrowers as a “targeted” subsidy, while we refer to subsidies that are at the full discretion of the lender as “non-targeted”. Because a non-targeted subsidy can be used to undercut contracts to profitable clients, the presence of such grants will thwart the realization of the Bertrand equilibrium when another MFI is unsubsidized.

**PROPOSITION 2:** The presence of a client-maximizing MFI with a non-targeted subsidy will prevent the entry, or force the exit of, any unsubsidized MFI.

**PROOF:** In order to maximize the number of borrowers in its portfolio, the client-maximizing MFI with a non-targeted subsidy minimizes the loss from capturing each borrower into its portfolio, $B^P \cup B^{NP}$. Since it is the NC rather than the PC with the moneylender that is binding under Bertrand competition, borrowers in $B^P$ require the smallest draw on the subsidy. Therefore, the subsidized MFI minimizes its loss on each borrower through a contract of $(V_i, r_i - \zeta)$ to $\forall i \in B^P$, where $\zeta$ is arbitrarily small. Such a contract lies on the Pareto efficient contract curve by **LEMMA 1**, and hence is preferred by borrower $i$ since $\Pi_i^P(V_i, r_i - \zeta) > \Pi_i^P(V_i, r_i - \zeta')$. Since MFIs are assumed to be identical in lending technology, a contract on the contract curve that yields $\Pi_i^P(V_i, r_i - \zeta)$ implies that $\Pi_i^{MFI}(V_i, r_i - \zeta) < 0$, therefore leaving no independently feasible contract for the unsubsidized MFI. □

**PROPOSITION 3:** Competition between two client-maximizing MFIs with non-targeted subsidies will lead to a Bertrand-Nash equilibrium in which the market share of each MFI will be proportional to its level of grant funding.

**PROOF:** See the Appendix
The intuition to the proof is that since Bertrand competition eliminates profits on each profitable borrower \( i \in B^p \), all borrowers must be captured through competitive subsidy in the client maximization process. One can think of each subsidized MFI as “purchasing” borrowers for its portfolio, where in equilibrium, arbitrage behavior takes place such that the “market price” of capturing each borrower is equilibrated across all those with access to MFI credit. The resulting Nash equilibrium can be seen in Figure 3 (with the example of \( G_1 > G_2 \)), where the specific borrower \( i \) served by each institution is undetermined. Note that and that all borrowers that remain in the portfolio \( B^p \cup \tilde{B}^{NP} \) are (weakly) better off than under cross-subsidization.

\[
\Pi_i (v_i^*, r_i^* (k))
\]

3.5 Competition from an MFI market entrant with a targeted subsidy

If one competitor is a client-maximizer with a targeted subsidy and the other a client-maximizer with a non-targeted subsidy, then the competitive equilibrium is different from the previous case since the targeting lender is prevented from expending grant resources on profitable clients. As a result, the non-targeting lender must only offer \( v_i^* - \zeta \) in order to capture all \( i \in B^p \). Since the non-targeting lender can capture \( i \in B^p \) at negligible cost, it is likewise able to focus (nearly) all of its grant resources on capturing as many \( i \in B^{NP} \) as possible. The nature of the grant-based competition is the same as the previous case, except only contracts to the poorer borrowers, \( i \in B^{NP} \), are contested.

**Proposition 3** now holds only over unprofitable borrowers; the share of \( \tilde{B}^{NP} \) captured by each lender now becomes proportional to grant funding. Thus, all \( i \in B^p \) are better off if neither lender targets, but
all \( i \in \mathcal{B}^\mathcal{NP} \) are better off if even one lender targets. (Note that only \( \mathcal{M} - 1 \) targeting lenders are required to make all \( \mathcal{M} \) client-maximizing lenders behave as if they were targeting.)

**PROPOSITION 4:** Market entry of a client-maximizing, subsidized MFI with funding specifically targeted to poor borrowers may cause poor borrowers to lose access to MFI credit.

**PROOF:** Recall that \( A \equiv \sum_{i \in \mathcal{B}^P} \Pi_i^{MFI} \) or the total profits realized on profitable borrowers by the incumbent client-maximizing MFI facing only moneylender competition (equal to the area \( A \) in Figure 2).

**Case (a):** The incumbent is unsubsidized. As the entrant (MFI2) and incumbent MFI1 engage in Bertrand competition over \( i \in \mathcal{B}^P \), \( A \to 0 \). In the Bertrand Nash equilibrium, the total subsidy for lending to the set of poorer borrowers \( \mathcal{B}^\mathcal{NP} \) will be smaller unless \( G_2 \geq A \). If and only if \( G_2 > A \), then \( \tilde{k}(0) < \tilde{k}(G_2) \), which implies \( \exists i \) for which \( i \in \mathcal{B}_1^\mathcal{NP} \), but \( i \not\in \mathcal{B}_2^\mathcal{NP} \), where \( \mathcal{B}_1^\mathcal{NP}, \mathcal{B}_2^\mathcal{NP} \subset \mathcal{B}^\mathcal{NP} \) are the sets of non-profitable borrowers captured before and after entry of the competing MFI. Any \( i \) for which \( i \in \mathcal{B}_1^\mathcal{NP} \) but \( i \not\in \mathcal{B}_2^\mathcal{NP} \) is strictly worse off since \( \Pi_i^B(V_{i^m}, r_{i^m}) > \Pi_i^B(V_{i^*}, r_{i^*}) \) by PROPOSITION 1.

**Case (b):** The incumbent has a targeted subsidy. (This case is a hybrid between the unsubsidized competitive equilibrium and the client-maximizer with an untargeted subsidy from section 3.5.) Without competition from the entrant MFI, the PC is binding for \( \forall i \in \mathcal{B}^\mathcal{NP} \) by LEMMA 2, and the total subsidy needed to capture each \( i \in \mathcal{B}_1^\mathcal{NP} \) into the portfolio is \( \sum_{i \in \mathcal{B}^\mathcal{NP}} ||\Pi_i^{MFI}(V_{i^*}, r_{i^*})|| = A + G_1 \). Consider entry by a client-maximizing MFI with targeted subsidy \( G_2 \). The ensuing competitive Nash equilibrium is similar to that in PROPOSITION 3 in that \( \Pi_i^{MFI} = \Pi_j^{MFI} < 0 \) for \( i \in \mathcal{B}_2^\mathcal{NP} \) except that the targeted nature of the subsidies results in \( \Pi_i^{MFI} = 0 \) for \( \forall i \in \mathcal{B}^P \). In contrast to case without MFI competition, the PC binds in competitive equilibrium for \( i \in \mathcal{B}^\mathcal{NP} \) only for the poorest borrower, \( i_{k(G_1 + G_2)} \), and the total MFI subsidy allocated to \( \mathcal{B}^\mathcal{NP} \) equals \( m_1^* + m_2^* ||\Pi_i^{MFI}| = G_1 + G_2 \), \( i = i_{k(G_1 + G_2)} \). (Refer to the proof of PROPOSITION 3). Thus for \( \mathcal{B}_2^\mathcal{NP} \supseteq \mathcal{B}_1^\mathcal{NP} \), we must have \( G_2 \geq m_1^* \cdot \Pi_i^{MFI} - G_1 \), where \( i = i_{k} \), otherwise it must be that \( \mathcal{B}_2^\mathcal{NP} \subset \mathcal{B}_1^\mathcal{NP} \). Observing that \( m_1^* \cdot \Pi_i^{MFI} - G_1 > A \), note that the conditions for the poorest borrowers losing access to MFI credit are less restrictive in Case (b) than in Case (a).

If \( G_2 \) indeed falls below the critical level shown in Case (b), then there are both winners and losers within \( \mathcal{B}^\mathcal{NP} \) under competition between MFIs with targeted subsidies. The set of borrowers \( \mathcal{B}_2^\mathcal{NP} \) are better off since competition to attract them into the portfolios increases the subsidy level of their
equilibrium contract. In contrast, those in $\tilde{B}_1^{NP}$ but left out of $\tilde{B}_2^{NP}$ are worse off since upon being dropped from the MFI portfolio, their profits fall from $\Pi_i^B(V_i^*, r_i^*)$ to $\Pi_i^B(V_i^{m*}, r_i^{m*})$.

An interesting implication of PROPOSITIONS 3 and 4 is that a maximum number of borrowers could be reached under a *collusive* strategy between the two MFIs. In a collusive strategy, MFIs could agree to split the market by jointly offering contracts at which the PC with the moneylender is binding. By PROPOSITION 3, the collusive strategy is not a Nash equilibrium. However, collusion generates a total subsidy of $A + G_1 + G_2$ to capture borrowers in $B^{NP}$, resulting in a larger set of poor borrowers, $\tilde{B}_2^{NP}$, with MFI credit access. Thus for any given $G_2$ we have $\tilde{B}_2^{NP} \subset \tilde{B}_1^{NP} \subset \tilde{B}_2^{NP}$, noting, however, that $\tilde{B}_2^{NP} \subset \tilde{B}_2^{NP}$ would have less favorable contracts under collusion than competition. The difficulties of sustaining collusion in this model are typical of any cartel, especially for loans to profitable borrowers. Nevertheless, it is interesting that collusion between client-maximizing lenders leads to the maximum redistribution towards the poor and the most evenly distributed benefits of microlending.

3.6 Summary of Conclusions from the Basic Competitive Model:

- Entry of an MFI is beneficial to all borrowers accessing MFI credit; it increases borrower profits from the optimal contract for the moneylender to the reservation profit of the moneylender.
- In competitive markets, profit-maximizers and unsubsidized client-maximizers always behave the same way. Thus, under competition it is not the motivation, but rather the extent and the nature of the grant funding of a lender that matters.
- Targeting of subsidies is unimportant in a market with a single client-maximizing MFI; the distinction only becomes important under MFI competition.
- Lenders with non-targeted subsidies can always drive any unsubsidized competitor out of the market altogether, whereas targeted subsidies can never eliminate a competitor from the market.
- Every competitive scenario involving a lender with targeted subsidies results in a market that is both competitive and in which some of the poor receive loans.
- Competition never makes any profitable borrower worse off.
- The only way in which the poor can be reached without subsidies is if a client maximizer exists as a solitary MFI in the market and competes only with a moneylender. However, in this case subsidies are merely being generated from amongst the other, less poor borrowers.
• If rents to the incumbent MFI from lending to the non-poor are higher than the grant funding of an entrant MFI, then the entrant can actually decrease the funding available to underwrite loans to the poor even if the grants are targeted specifically towards the poor.
• Collusive behavior between client-maximizing MFIs can result in the maximum number of borrowers reached by MFIs, and the most evenly distributed benefits among borrowers.

4. Extended Model: Asymmetric Information between Lenders

4.1 Asymmetric information and dynamic incentives

In the previous section, we developed a basic model in which information about borrower heterogeneity was common knowledge between borrower and lender. An implicit assumption of the basic model is that a lender is able to ascertain the optimal contract through observation of a borrower’s capital endowment. We continue to assume that heterogeneity in $\mathcal{K}$ remains observable to the MFI. However, we now assume that each borrower $i \in \mathcal{B}$ is characterized by a personal rate of time preference, $\rho_i \in [\rho, \overline{\rho}]$ per lending period, information that is hidden to the MFI, whose per-period profit function we assume is unchanged and homogeneous across institutions. Let $g(\rho_i)$ represents the density function of $\rho_i$ and $G(\rho_i)$ its associated distribution function. The distribution of time preference is assumed to be orthogonal to the distribution of $k$. Timing is more important in the extended model, so we lay it out explicitly as follows:

| Borrower endowed with fully observable $k_i$ and $\rho_i$ observable only to borrower. | Borrowers apply for loans | Screening by lender occurs. Borrowers with other loans denied new credit. | Capital provided to borrower and invested | Returns realized on investment project; lender repaid fully or partially |

**Figure 4**

We now bring into our analysis the issue of dynamic incentives. Dynamic incentives provide motivation for repayment when borrowers lack collateral to secure loans by implicitly promising
continued credit access as a reward for loan repayment. They are routinely used by MFIs (and other lenders) in poor areas of developing countries to mitigate issues of moral hazard involved with credit transactions. The present value to a borrower of the continued access to MFI credit in each period, \( \Gamma(k_i, \rho) \), is positively related to the advantage offered by MFI financing relative to the alternative (moneylender) contract, and a negatively related to a borrower’s rate of time preference.\(^{10}\)

To focus our attention on issues related to problems of asymmetric information and competition, we will concentrate on the case of two MFIs engaged in Bertrand competition over the pool of borrowers with \( i \in \mathcal{B}^P \) (Competition over \( i \in \mathcal{B}^{NP} \) yields similar results.) Hoff and Stiglitz (1998) show that dynamic incentives are weakened by new market entrants as this improves the reservation loan contract available to borrowers in the case of default. Their results imply borrowers must somehow be punished for default by the financial system as a whole, through a system of negative borrower information-sharing, \( i.e. \) each lender sharing its \textit{lista negra} (as it is often referred to in Latin American MFIs--the blacklist).

What we illustrate in this section is that even within a system that identifies defaulting borrowers, other gaps involving asymmetric information between lenders must be bridged. We show that it is also critical for lenders to share \textit{positive} borrower information with one another, even regarding well-performing loans, \( i.e. \) that a \textit{lista blanca} (a list of positive information) is also necessary. This is true even in a model without strategic default. In order to concentrate on this issue, we take as our informational benchmark market with Bertrand competition in which all lenders fully share the \textit{lista negra}, where defaulters are denied future formal credit access, but no positive information is shared.

Calculating the present value of the relationship with the MFI, the objective function of borrower \( i \) under dynamic incentives now becomes

\[
\max \Pi_i = (1 - \rho_i) \left[ \beta - (1 + r_i) \right] \nu_i' + \Gamma(k_i, \rho_i).
\]

\(^{10}\) To concentrate our analysis on borrower behavior we assume that lenders continue to maximize single-period profits; the results derived are consistent with lender discounting provided that all lenders share the same discounting process.
As $\rho_i$ increases, the borrower becomes less concerned about lack of future credit access, and with any $k_i \in \mathcal{K}$, prefers a larger loan size. We see this through total differentiation of (6) with respect to $V_i$ and $r_i$ which yields

$$
\frac{dV_i}{dr_i} = -\frac{V_i(1 - p_i)}{(\beta - (1 + r_i))(\beta - \beta_i) - p_i \Gamma(k_i, \rho_i)}
$$

(7)

Notice as $\rho_i$ increases, the borrower’s indifference curve rotates clockwise, with the upward-sloping part of the curve becoming flatter and the downward sloping part becoming steeper as in Figure 5. This implies that, for example, in a Bertrand equilibrium for $\rho_2 > \rho_1$

$$
V_i^{**}|_{\rho_2 > V_i^{**}}|_{\rho_1} \quad \text{and} \quad r_i^{**}|_{\rho_2 > r_i^{**}}|_{\rho_1}
$$

(8)

or that the equilibrium contract for a more impatient borrower is characterized by a higher loan size and interest rate than the contract for a more patient borrower. Thus an MFI operating in the context of information sharing between lenders about borrower indebtedness does not face a hidden information problem; it can infer a borrower’s impatience from $k_i$ and the requested loan size. As $\rho_i$ varies for any given $k_i$, it creates a unique tangency point on the isoprofit curve of the lender at a unique level of $V_i$ for any given $k_i$, i.e. $V_i^*(k_i) = \frac{1}{2p_i} \left[ \frac{\beta - (1 + c) - p_i \Gamma(k_i, \rho_i)}{\beta - \beta_i} - \beta - p_i k_i \right]$. As Figure 5 shows, with full information sharing, an MFI could make up for the increased likelihood of default on this larger loan size by offering even the most impatient borrower a larger loan at a higher interest rate.

However, when there are multiple lenders and an absence of information sharing about borrower indebtedness between them, there is an incentive for borrowers past a certain level $\rho_i = \rho^*$ to take multiple loans, where we assume $\rho^* \in (\underline{\rho}, \bar{\rho})$. Previous work typically adds the prevention of multiple contracting as a constraint for the principal; instead we take the approach of seeing how multiple contracting emerges without safeguards, and how the sharing of information can be used to prevent it.11

11 The problem of multiple contracting in markets with asymmetric information is not unique to microfinance; see Cawley and Philipson (1999) for an application to life-insurance markets, and Kahn and Mookherjee (1998) for a more general model of non-exclusive contracting in credit markets.
Let $\rho^*$ be the lowest value of $\rho_i$ that satisfies the following inequality, in which expected profits from multiple loans are higher for the borrower than on a single loan:

$$\Pi_i^\beta = \alpha \cdot \varepsilon / \rho_i + (1 - \alpha) \left[ (1 - p_i) \left( \beta (1 + \tilde{r}_i) \right) \tilde{V}_i + \Gamma(k_i, \rho_i) \right] > (1 - p_i) \left( \beta (1 + \tilde{r}_i) \right) \tilde{V}_i + \Gamma(k_i, \rho_i)$$ (9)

where $\alpha$ represents an information parameter such that an MFI is able to identify a borrower’s true level of indebtedness with probability $\alpha$ before a borrower receives a loan. Unless there are institutional arrangements for information-sharing between lenders, field experience has shown that $\alpha$ typically falls as the number of MFIs in a given market increases.

The difference in behavior between more patient and less patient borrowers is illustrated graphically in Figure 5 where, for example, the most patient borrower with $\rho = \rho_i$ receives an equilibrium contract for loan of size $\tilde{V}_i(k_i)$ at interest rate $\tilde{r}_i(\tilde{V}_i(k_i))$. Under full information sharing, the less patient borrower with $\rho = \rho^*$ receives an equilibrium contract for a loan $2\tilde{V}_i$ at the higher interest rate $\tilde{r}_i(2\tilde{V}_i)$. A single lender is thus willing to oblige such a borrower by providing a larger loan along its zero-profit offer curve at a higher interest rate that accounts for the higher expected rate of default.

This is the contract that will exist in the absence of information asymmetries under exclusive contracting between borrower and lender.

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12 We clarify three additional simplifying assumptions in the extended model: borrowers discovered with multiple loans are culled from the portfolio, a borrower who defaults on any loan defaults on all loans, and a borrower who defaults on an MFI loan is henceforth able to access credit only from the moneylender.
With an absence of information-sharing between lenders, however, the borrower with \( \rho_i \geq \rho^* \) can lower his overall cost of borrowing by obtaining two separate loans from two different lenders, while creating the illusion for each lender that he is borrowing a fraction of his actual total—a ubiquitous occurrence as reported by MFIs operating in competitive markets. In the example in Figure 5, the less patient borrower with \( \rho^* \) benefits by obtaining two loans from two different lenders of size \( \bar{V}_i(k_i) \), and paying interest rate \( \bar{r}_i(\bar{V}_i(k_i)) < \bar{r}_i(2\bar{V}_i(k_i)) \) on each of the two different loans, increasing his utility to \( \Pi'_B(\rho^*)' \). Moreover, the lower interest rate sensitivity on the joint offer curve \( \Pi^MFI_i(k_i) + \Pi^MFI_2(k_i) = 0 \) faced by the multiple-borrower induces the borrower to increase borrowing to \( 2\bar{V}_i(k_i) > 2\bar{V}_i(k_i) \) by taking two loans of size \( \bar{V}_i(k_i) \) at interest rate \( \bar{r}_i(\bar{V}_i(k_i)) \), further increasing the impatient borrower’s utility from \( \Pi'_B(\rho^*)' \) to \( \Pi''_B(\rho^*)'' \). Multiple contracting occurs in the absence of full information because the lender can no longer infer \( \rho_i \) from the requested loan size. The lender cannot distinguish between an impatient borrower taking two separate loans of the size a patient borrower with the same \( k_i \) would demand. Note in our example that instances of multiple contracting will occur whenever the full-information equilibrium \( V_i \) for the least patient borrower is two or more times greater than the equilibrium contract for the most patient borrower for any \( k_i \).\(^{13}\)

This behavior creates an externality that raises the overall default rate for MFIs for two reasons: First, because the interest rate is lower under multiple borrowing than it would have been with a single lender, total borrowing and indebtedness increases within the portfolio of borrowers. Second, and more

\[^{13}\] The result is generalizable to a borrower taking more than two separate loans, given sufficient borrower impatience and market competitors.
importantly to an individual MFI, the true probability of default is now a function of its own lending
and of some unknown quantity borrowed from elsewhere.

This makes the expected default rate of any given borrower $i$ now equal to

$$
\hat{p}_i = \frac{\int p(V_i)g(\rho_i)d\rho_i + (1-\alpha)\int p(2\tilde{V})g(\rho_i)d\rho_i}{\int g(\rho_i)d\rho_i - \alpha \int g(\rho_i)d\rho_i} \quad (10)
$$

Defining $\gamma \equiv 1 - G(\rho^*)$ as the probability of multiple contracting, and letting $\tilde{p}_i(k_i)$ and $\tilde{p}_i(k_i)$ equal expected probabilities of default for borrowers (at a given level of $k_i$) with single and multiple loans respectively, we obtain

$$
\hat{p}_i \equiv p_i(k_i, V_i, \alpha) = \frac{(1-\gamma)\tilde{p}_i + \gamma(1-\alpha)\tilde{p}_i}{1-\gamma\alpha}, \quad (11)
$$

noting that $\gamma = \gamma(\alpha)$. Since an increase in $\alpha$ makes (9) less likely to be satisfied, $\frac{d\rho^*}{d\alpha} > 0$ and so $\gamma_\alpha < 0$. (Note since $G(\rho_i)$ and $\alpha$ are both assumed to be independent of the distribution of $k$, we know $\gamma$ and $\gamma_\alpha$ are independent as well.) As seen in (11), as $\alpha \to 1$, $\tilde{p}_i \to \tilde{p}_i$, or the default rate approaches the full information case. Using (2) and (11), expected profits to the lender for borrower $i$ now become

$$
\Pi_i^{MFI} = (1-\tilde{p}_i)(r_i - c)V_i - \tilde{p}_i(V_i(1+c-\beta) - F) = \frac{(1-\gamma)\tilde{\Pi}_i^{MFI} + \gamma(1-\alpha)\tilde{\Pi}_i^{MFI}}{1-\gamma\alpha}, \quad (12)
$$

where $\tilde{\Pi}_i^{MFI} = (1-\tilde{p}_i)(\tilde{r}_i - c)\tilde{V}_i - \tilde{p}_i(1+c-\beta)\tilde{V}_i - F$ and $\tilde{\Pi}_i^{MFI} = (1-\tilde{p}_i)(\tilde{r}_i - c)\tilde{V}_i - \tilde{p}_i(1+c-\beta)\tilde{V}_i - F$. Equation (12) yields the information necessary for Proposition 5:

**PROPOSITION 5:** If asymmetric information between lenders increases as the number of competing lenders increases, borrowers receive less favorable loan contracts after entry of new lenders.

**PROOF:** The details of the proof are given in the Appendix. However, first note the positive relationship between profits to each MFI and $\alpha$. The Appendix shows that partial differentiation of (12) yields
\[
\frac{\partial \Pi_i^{MFI}}{\partial \alpha} = \left[ \gamma (1 - \gamma) - \gamma_{\alpha} (1 - \alpha) \right] \frac{\left( \Pi_i - \Pi_i^0 \right)}{(1 - \gamma \alpha)^2} > 0.
\] (13)

Under Bertrand competition, each lender will adjust the contract to borrower \( i \in B^p \), so the resulting equilibrium contract yields \( \Pi_i^{MFI} = 0 \). If new entrants make information-sharing more difficult, total differentiation of (12) reveals that each lender’s new zero-profit curve must lie strictly to the right and to the interior of the old one. The lower-information equilibrium with more lenders in the market lies on a lower isoprofit function for the borrower at which any size loan, offered at zero-profit to the lender, is offered at a higher interest rate. The conclusion from the proof is illustrated in Figure 6, which shows the change in the Bertrand competitive equilibrium as \( \alpha \) falls.

Once the competing MFIs respond to multiple contracting by adjusting equilibrium contracts for all clients, it is unclear whether or not the impatient have indeed benefited. The interest rate “discount” received though multiple loan contracting may or may not compensate for the fact that every individual loan contract is marginally worse. What is unambiguous is that patient borrowers with \( \rho_i < \rho^* \), who find it optimal to borrow only from a single lender, have been hurt by reduced informational flows between lenders, and the ensuing instances of multiple contracting by other borrowers. By undertaking action clearly observable to only one lender, the impatient create a classical externality whose costs are spread across the whole population of borrowers.
4.2 Effect of asymmetric information on the poorest borrowers

**PROPOSITION 6:** As asymmetric information between lenders increases, the poorest borrowers are dropped from the lending portfolio.

**PROOF:** Consider the poorest borrower \( i \in B^p \) who receives a loan with under Bertrand competition when

\[
\Pi_i^{MFI}(r_i^*, r_i^*)|_{\alpha} = (1 - \hat{p}_i(\hat{k}_i))r_i - c V_i - \hat{p}_i(\hat{k}_i) V_i (1 + c - \beta) - F = 0.
\]

Total differentiation yields

\[
\frac{d\hat{k}_i}{d\alpha} = -\frac{\hat{p}_i}{\hat{p}_k}, \text{ where } \hat{p}_k < 0 \text{ by assumption.}
\]

We know that

\[
\hat{p}_i = \frac{\hat{p}}{\gamma(1 - \gamma) - \gamma(1 - \alpha)} \frac{(\hat{p} - \hat{p})}{(1 - \gamma\alpha)^2} > 0
\]

by differentiation of (11). Thus

\[
\frac{d\hat{k}_i}{d\alpha} = -\frac{\hat{p}_i}{\hat{p}_k} < 0, \text{ or as } \alpha \text{ declines to } \alpha', \text{ the poorest borrower who receives an MFI loan must have initial assets } \hat{k}_i + \psi > \hat{k}_i. \quad \Box
\]

The intuition to the proof is straightforward. Increasing information asymmetries increase the cost of lending to the entire portfolio of borrowers. Consider the poorest borrower receiving MFI credit under Bertrand competition at a given level \( \alpha \) of information sharing, the borrower who has initial assets \( \hat{k}_i \).

As informational flows decline with an increasing number of lenders in a market to \( \alpha' < \alpha \), the probability of default increases by \( \hat{p}_i \) such that marginally profitable loans to borrowers with level of initial assets \( \hat{k}_i \) (and slightly above \( \hat{k}_i \)) become unprofitable. As a consequence, the dilution of information between lenders resulting from the entry of new MFIs in a given area, even non-profit institutions, may cause the poorest borrowers to be dropped from MFI portfolios.

4.3 Conclusions from extended model with asymmetric information:

- With asymmetric information between competing MFIs, every loan contract yields a lower profit to the borrower than under the full information benchmark. Patient borrowers are always worse off with reduced information sharing. It is possible that the impatient borrowers are worse off as well, if the negative externality of multiple contracting overwhelms the direct benefit.
• If asymmetric information between lenders increases with the number of MFIs in the market, competition has an unambiguously negative effect on both the most poor and the most patient borrowers in the portfolio.

• Optimal information sharing between lenders must include not only data on defaulting borrowers, the *lista negra*, but also continually updated information on current borrowers, even those who are not defaulting, or the *lista blanca*.

5. Policy Implications

It seems intuitive that the dramatic growth in the number of MFIs in developing countries, and the ensuing competition between them, would have an unambiguously positive effect on low-income entrepreneurs in developing countries. However, this research presents number of hypotheses that provide reason to question this notion.

We believe that a number of policy conclusions flow from this research. At the broadest level, the results of the first part of our paper may extend to other instances in which altruistic motivation induces a non-profit institution to cross-subsidize. Examples may include medical or health services, socially motivated education programs, or provision of low-income housing, in which there exists some degree of competition between for-profit and non-profit entities.

More specifically to microfinance, our research implies that the structure of funding is unimportant in monopolistic markets, whereas the motivation of lenders is less important in competitive markets. Therefore, as competition increases, the onus for the inclusiveness of the market passes from the practitioners of microfinance to the donors. Yet the very existence of competitive markets hinges on the idea that grant funding be used in a competitive market only to subsidize the cost of lending to the poor. In light of this, our research supports the notion put forth by Morduch, that financially self-sufficient MFIs should co-exist with their subsidized counterparts, provided that these subsidies are carefully restricted to the poorest borrowers.
Targeting of subsidies is difficult to achieve in practice, as such constraints are hard to monitor, and MFIs themselves may not be able to easily identify the dividing line between profitable and unprofitable clients. In light of these issues, it may be most practical to reserve grants strictly for geographical expansion of MFI activity into poor, unserved areas. In this way, donors can avoid undermining competitive markets without placing expensive, ill-defined restrictions upon practitioners. It also may be possible to restrict grant funding to methodologies that do not appeal to profitable borrowers, such as joint liability lending contracts where loan amounts are typically small and organizational costs are high.

Another clear implication from our research is the need for credit bureaus, or internet-based central risk-management systems, which identify outstanding debt in addition to cases of default. In general the astounding growth in MFI lending in many areas has vastly outpaced the ability of MFIs to monitor borrower quality and indebtedness. Among the Central American countries, for example, there remains great heterogeneity in informational infrastructure. While some countries such as El Salvador have established reasonably well-functioning centralized risk-management structures, others such as Nicaragua lag far behind in this area, although the density of MFI activity is extremely high. At this stage in the microfinance movement, the establishment of such centralized risk networks must become a leading priority to ensure the success and sustainability of the microfinance movement in LDCs.
Appendix

EXPLICIT FORMULATION OF ASSUMPTIONS AND BOUNDARY CONDITIONS:

Assumption 1: “Moneylender Feasibility”. Since the maximum interest rate acceptable to a borrower lies very close to $\bar{\beta} - 1$, we can approximate the condition for the least favorable loan to a borrower that leaves zero profit to the moneylender as $\Pi_i^{ml} = (1 - p_i)\bar{\beta}V_i + p_i\beta V_i - (1 + c^m)V_i = F^m$. Substituting the expression for $V_i^*(k_i)$ in (5b) into the moneylender zero-profit condition, and solving for $k$ shows that for moneylender feasibility $k$ must be

$$k > \frac{1}{p_k} \left[ \bar{\beta} - \frac{2p_iF^m}{\bar{\beta} - \beta} - \frac{\beta - (1 + c^m)}{\beta - \beta} \right].$$

Assumption 2: “Microfinance Lending Environment”. Lemma 2 shows that MFI profits in competitive equilibrium can be expressed as $\Pi_i^* = \frac{1}{p_i} \left[ (\Delta - p_i)^2 - (\Delta^m - p_i^m)^2 \right] (\bar{\beta} - \beta) + F^m - F$, where

$$\Delta = \frac{\bar{\beta} - (1 + c_i)}{\beta - \beta} \quad \text{and} \quad \Delta^m = \frac{\bar{\beta} - (1 + c^m)}{\beta - \beta}.$$  Setting this expression equal to zero and substituting in $p_i = \overline{\beta} + p_kk_i + p_iV_i$ shows that the equilibrium break-even level of initial borrower assets for the MFI is

$$\hat{k} = \frac{-1}{p_k} \left[ \overline{\beta} + \frac{2p_i(F - F^m)}{c^m - c} - \frac{\Delta^* + \Delta^m}{2} \right].$$  By setting the above boundary condition in Assumption 1 for $k$ less than $\hat{k}$ and solving for parameter conditions, we find that an unprofitable borrower will exist provided that $\sqrt{2p_i(\sqrt{\overline{\beta} - \sqrt{F^m}})} > \frac{c^m - c}{\sqrt{\bar{\beta} - \beta}}$, which holds given sufficiently high $\bar{\beta}$. Provided that $\overline{k} > \hat{k} + \xi$, more than one profitable borrower will exist, which satisfies the 2nd part of Assumption 2.

Boundary Conditions on Default Rate: Here we delineate parameters for the model to ensure that $0 < p(V_i, k_i) < 1$ for $\forall k_i \in [\overline{k}, \hat{k}]$. Substituting $V_i^*(k_i) = \frac{1}{2p_i} [\Delta - \overline{\beta} - p_kk_i]$ into the default function yields $p_i(k_i) = \frac{1}{2} (\overline{\beta} + \Delta + p_kk_i)$. Recalling that $p_i < 0$, our model parameters must ensure $p(k) < 1$ and $p(\overline{k}) > 0$. The conditions are thus that $k > \frac{1}{-p_k} (\overline{\beta} + \Delta - 2)$ and $\overline{k} < \frac{1}{-p_k} (\overline{\beta} + \Delta)$. The former always holds since $\overline{\beta} + \Delta - 2$ is always $< 0$. The latter condition that $\overline{k} < \frac{1}{-p_k} (\overline{\beta} + \Delta)$ is consistent with the
boundary condition in Assumption 2 provided

\[\frac{1}{-p_k} (\bar{p} + \Delta) > -\frac{1}{p_k} \left[ \bar{p} + \frac{2p_k(F - F^m)}{c^m - c} - \frac{\Delta + \Delta^m}{2} - \xi p_k \right],\]

or

\[3\Delta + \Delta^m + 2p_k \xi > \frac{4p_k(F - F^m)}{c^m - c},\]

which holds given sufficiently high \(\bar{p}\) relative to \(p_v\).

Proof of Lemma 1: We show in turn that (i) the constraint set impose by any PC (given as a reservation level of borrower profit) is compact and convex; (ii) the contract that maximizes lender profit is unique for every PC; (iii) that (i) and (ii) yield a set of points that form a Pareto-efficient contract curve for each borrower and lender by the assumptions of the model.

(i) Assumption 1, which says that every borrower in the pool is offered some contract by a monopolistic moneylender, ensures that a PC exists and is defined by some level of borrower profit \(\Pi_i^B\) for each borrower \(i\). Re-arranging the borrower’s profit function in (1) and solving for \(r_i\) at any (non-negative) constant level of profit of \(\Pi_i^B\), yields

\[r_i = \bar{\beta} - \frac{\Pi_i^B}{V_i(1 - \bar{p} - p_k k_i - p_v V_i)} - 1.\]

Differentiation gives

\[\frac{dr_i}{dV_i} = \frac{\Pi_i^B}{V_i^2 \left(1 - \bar{p} - p_k k_i - p_v V_i\right)} \cdot \frac{V_i^2 (1 - \bar{p} - p_k k_i - p_v V_i) - (1 - \bar{p} - p_k k_i - 2p_v V_i)^2}{V_i^4 (1 - \bar{p} - p_k k_i - p_v V_i)^4} < 0.\]

Thus, the borrower’s isoprofit curves are concave and decreasing in \(r_i\), which implies that the upper contour set above any isoprofit contour is a convex set. Setting \(r_i = 0\) for the borrower’s profit function in (1) and solving for \(V_i\), note that the each isoprofit curve of the borrower, for any \(\Pi_i^B \geq \varepsilon\), intersects the \(V_i\) axis at the positive values \(V_H^i\) and \(V_L^i\)

\[V_H^i = \frac{1}{2p_v} \left(1 - p_i - p_v k_i\right) \pm \sqrt{\left(1 - p_i - p_v k_i\right)^2 - 4p_v^2 \left(\frac{\Pi_i^B}{\bar{\beta} - 1}\right)}.\]

That the constraint set is closed follows from the observation that (a) \(\Pi_i^B(V_i, r_i)\) is a continuous function; and (b) the constraint set is the intersection of \(\{(V_i, r_i) \mid V_L^i \leq V_i \leq V_H^i \text{ and } r_i \geq 0\}\) and the inverse image, under \(\Pi_i^B\), of the set \(\Pi_i^B \mid \Pi_i^B \geq \Pi_i^B(V_i, r_i)\). Boundedness follows from the fact that the constraint set lies at and below the isoprofit curve defined by (1) within and including the positive values \(V_H^i\) and \(V_L^i\), together with non-negativity constraint that \(r_i \geq 0\). Since all inequalities are weak, the constraint set is bounded. Therefore, the constraint set is compact and convex, and a maximum exists.
(ii) The properties of the lender’s isoprofit contours can be similarly derived re-arranging (2) while holding lender profits constant and solving for \( r_i \), yielding

\[
\frac{dr_i}{dV_i} = \frac{p_v V_i^2 (1+c-\beta)-(\Pi^L_i + F^L_i)(1-p_i-p_v V_i)}{V_i^2 (1-p_i)^2},
\]

and

\[
\frac{d^2 r_i}{dV_i^2} = 2 p_v^2 V_i^4 (1-p_i)(1+c-\beta) + 2 V_i(\Pi^L_i + F^L_i)(1-p_i)[(1-p_i-p_v V_i)(1-p_i)+p_v^2 V_i]}
\]

Since for the borrower and lender isoprofit curves \( \frac{dr_i}{dV_i} < 0 \) and \( \frac{dr_i}{dV_i} > 0 \) respectively for

\[
V_i > (1-\bar{p} - p_k k_i)/2p_v \quad (i.e. \quad (1-p_i-p_v V_i) < 0),
\]

a sufficient condition to establish a lender maximum at the tangency point to \( \Pi^B_i \) is that \( \Pi^L_i + F^L_i \geq 0 \), which is true at \( V^*(k_i) \) by Assumption 3, it is easily shown that for sufficiently large \( p_k, \Pi^L_i + F^L_i \geq 0 \). Thus the lender’s isoprofit curve is increasing and convex over \( V_L \leq V_i \leq (1-\bar{p} - p_k k_i)/2p_v \) and hence at \( V^*(k_i) \). That the tangency point \( (V^*_i, r^*_i) \) defined by \( V^*(k_i) \) at each level of \( \Pi^B_i \) and \( i \in \mathcal{B} \) is unique is shown by equation (5b) and the fact that \( r_i(V_i) \) is one-to-one over the domain \([V_i, V=(1-\bar{p} - p_k k_i)/2p_v]\). We know that the optimal \( V^*(k_i) \) that defines each horizontal contract curve lies within \([V^L, V^H]\) (and thus satisfies the non-negativity constraints) for \( \forall k_i \in [k, \bar{k}] \), since re-arranging (5b) yields the

condition

\[
k > \frac{1}{1-p_k}(\bar{p} - \Delta),
\]

which satisfies the boundary condition that

\[
k \geq \frac{-1}{p_k} \left[ \bar{p} + \sqrt{\frac{2p_v F^m}{\beta - \beta}} - \Delta \right].
\]

Assumption 1. Furthermore, note that \( V^*(k_i) \) can never exceed \( V^H \) since \( V^*(k_i) < (1-\bar{p} - p_k k_i)/2p_v \).

(iii) Assumption 1 guarantees that the PC (and thus a contract curve) is defined for all \( i \in \mathcal{B} \) with each lender. That the locus of points on the contract curve are Pareto-efficient can be proven by contradiction: If a point on the contract curve were not Pareto-efficient, another contract would exist for which \( \Pi^L_i (V_i, r_i) \) could be increased holding \( \Pi^B_i (V_i, r_i) \) constant. But then this point could not have been a maximum of the objective function, and hence could not be on the contract curve. □

Proof of LEMMA 2: We prove each part of the LEMMA sequentially:

(i). Partial differentiation of equation (5b) shows that

\[
\frac{\partial V^*_i}{\partial k_i} = \frac{\partial V^m_i}{\partial k_i} = \frac{-p_k}{2p_v} > 0.
\]
(ii). To show that MFI profits in competition with a moneylender are higher for higher $k_i$, we solve explicitly for the equilibrium MFI loan as a function of parameters. First, we utilize the fact that the zero-profit constraint of the moneylender will bind for all loans under competition. Hence we can solve for the equilibrium rate of interest charged by the moneylender, $r^{m*}_i$ for $\forall i$ in the MFI portfolio. Letting $p^m_i$ be the borrower’s probability of default to the moneylender and where again $\Delta^m = \frac{\bar{\beta} - (1 + c^m)}{\beta - \beta}$, then $V^m_i = \frac{\Delta^m - p^m_i}{p_v}$, so the zero-profit interest rate will be $r^{m*}_i = \frac{\frac{p_v F^m}{\Delta^m - p^m_i} - p^m_i \beta + (1 + c^m)}{1 - p^m_i} - 1$.

In the continuous equilibrium, the PC will bind such that $\Pi^P_i(V^{m*}_i, r^{m*}_i) = \Pi^P_i(V^*, r^*)$ for all borrowers. (To see this, suppose it did not; by increasing $r_i$ to the point where the PC binds, the MFI could retain that borrower in the portfolio $B^P \cup \tilde{B}^{NP}$ and relax the BC. Therefore, the original contract could not have been an equilibrium.) We substitute $r^{m*}_i$ into the binding PC

$$(1 - p_i)(\Delta - p_i)(\bar{\beta} - (1 + r^{m*}_i)) = (1 - p^m_i)(\Delta^m - p^m_i)(\bar{\beta} - \frac{\frac{p_v F^m}{\Delta^m - p^m_i} - p^m_i \beta + (1 + c^m)}{1 - p^m_i})$$

and solve for the competitive equilibrium MFI rate of interest $r^{*}_i$ as a function of the parameters of the model, obtaining

$$r^{*}_i = \bar{\beta} - \frac{(\Delta^m - p^m_i)^2 (\bar{\beta} - \beta) - p_v F^m}{(1 - p_i)(\Delta - p_i)} - 1.$$  Substituting for $r^{*}_i$ and $V^{*}_i$, we can now solve explicitly for the equilibrium level of MFI profit with the PC and the moneylender zero-profit constraints binding,

$$\Pi^{MFI}_i = (1 - p_i) \left[ \frac{(\Delta^m - p^m_i)^2 (\bar{\beta} - \Delta - p_i)}{(1 - p_i)(\Delta - p_i)} \right] + p_i \beta - (1 + c) \left[ \frac{\Delta - p_i}{p_v} \right] - 
\frac{[(\Delta - p_i)^2 - (\Delta^m - p^m_i)^2](\bar{\beta} - \beta)}{p_v} + F_m - F.$$  

Taking the derivative of this equilibrium MFI profit level with respect to $k$, we get

$$\frac{\partial \Pi^{MFI}_i}{\partial k} = -2 p_k (\bar{\beta} - \beta)[(\Delta - p_i) - (\Delta^m - p^m_i)] \frac{1}{p_v}$$, which $> 0$ if $c^m - c > (p_i - p^m_i)(\bar{\beta} - \beta)$. Substituting
in the default function, \( p_i = \overline{p} + p_k k_i + p_v V_i \), into this expression, we can rewrite this inequality as
\[
-c^m - c > p_v (V_i - V_i^m)(\overline{\beta} - \beta).
\]
Solving explicitly for the optimal loan size as a function of \( k_i \),
\[
V_i = \frac{\Delta - \overline{p} - p_k k_i}{2p_v} \quad \text{and} \quad V_i^m = \frac{\Delta^m - \overline{p} - p_k k_i}{2p_v},
\]
so \( V_i - V_i^m = \frac{\Delta - \Delta^m}{2p_v} \). Hence the required condition for
\[
\frac{\partial \Pi_{MFI}^*}{\partial k} > 0 \quad \text{reduces to} \quad c^m - c > p_v (\overline{\beta} - \beta) \left( \frac{\overline{\beta} - (1 + c) - \beta + (1 + c^m)}{2p_v(\overline{\beta} - \beta)} \right) \quad \text{or} \quad c^m - c > \frac{c^m - c}{2}.
\]

By Assumption 2, \( \Pi^*(\overline{k}) > 0 \), and \( \Pi^*(\overline{k}) < 0 \). \( \Pi^*(k_i) \) is a continuous, increasing function of \( k \) because \( p \) is a continuous function of \( k \) and \( p_v \) is a positive constant, so by the Intermediate Value Theorem there exists some \( \hat{k} \), with \( \overline{k} < \hat{k} < \overline{k} \), for which \( \Pi^*(\hat{k}) = 0 \). We denote as the “break-even borrower” the individual \( \hat{i} \) whose capital endowment \( \hat{k}_i \) is the closest to \( \hat{k} \) while being \( \geq \hat{k} \); thus for the break even-borrower it will be the case that \( \Pi^*(k_i) \geq 0 \) but \( \Pi^*(k_{i-1}) < 0 \).

(iii). That MFI client-maximization implies cross subsidization from \( B^P \) to \( \tilde{B}^{NP} \) flows directly from the preceding arguments; the MFI maximizes its number of clients by maximizing profits for \( B^P \) and minimizing losses in capturing \( \tilde{B}^{NP} \), both subject to the PC, and hence uses profits from the former to cross-subsidize the latter. To show that it is able to do so, first note that
\[
\frac{\partial^2 \Pi_{MFI}^*}{\partial k^2} = -2p_k(\overline{\beta} - \beta)(p_k - p_k) = 0,
\]
so the profits of the MFI in equilibrium are an increasing and linear function of \( k \). Since we have assumed that \( \hat{k} \) refers to the borrower who weakly satisfies a zero-profit condition for the MFI under competition, \( \Pi_{MFI}^*(\hat{k}_i) \geq 0 \). The linearity of the profit function combined with the uniform distribution of \( k \) implies that \( \Pi^*(k_i) + \Pi^*(k_{i-1}) \geq -(\Pi^*(k_{i-1})) \), and since Assumption 2 tells us that there exist one unprofitable borrower and more than one profitable borrower, \( 1 < \hat{i} < n \), so \( k_{i+1} \) and \( k_{i-1} \) exist and thus cross-subsidization of at least \( k_{i-1} \) is possible.
(iv). Let \( S^+ = \sum_{i=1}^{n} \Pi_i^e(k_i) + G \) be the surplus profits from lending to profitable borrowers plus the subsidy received by the MFI. In addition, let \( S_m = S^+ + \sum_{j=1}^{m} \Pi_i^e(k_{i-j}) \) for \( m = 1, \ldots, \hat{i} - 1 \) be the finite, decreasing sequence of total profits from lending to successive borrowers below \( \hat{k} \). The maximum number of borrowers \( n^* \), and hence the equilibrium, will be reached when \( S_m \) is as close to zero as possible while remaining non-negative. Consider three cases: (a) If \( S_m = 0 \) for some \( m^* \), then the BC and the PCs all bind in equilibrium, and \( n^* = n - \hat{i} + m^* + 1 \). For simplicity of exposition we have assumed case (a) to be operative in our analysis. The alternative cases do not affect the fundamental insights of our model, but are nevertheless interesting. (b) If \( S_m > 0 \) for \( m^* \) but \( S_m < 0 \) for \( m^* + 1 \), then some small residual profit exists due to the discrete distribution of \( k_i \). In this case an infinite number of equilibrium contracts exist in which this residual profit may be distributed within the portfolio, and either the BC or some borrowers’ PCs do not bind, but \( m^* \) and hence \( n^* \) remain unchanged. (c) If \( S_m > 0 \) when \( n^* = n \), the MFI has surplus funds even after reaching the entire group of potential borrowers. Note, however, even in this case \( n^* = n \) remains unique regardless of the rule used to dispose of excess profits. Thus for any given MFI subsidy \( G \geq 0 \), \( \forall i \in [1, \hat{i} - m^* - 1] \) receive the monopolistic moneylender contract \((V_i, r_i) = (V_i^m, r_i^m)\), and no loan contract from the MFI, and \( \forall i \in [\hat{i} - m^*, n] \) receive the MFI loan contract \((V_i, r_i) = (V_i^*, r_i^*)\) and no loan from the moneylender.

**Proof of Proposition 3:** Consider competition between two client-maximizing MFIs, \( h = \{1, 2\} \) with non-targeted subsidies \( G_1 \) and \( G_2 \), respectively. A best response by MFI \( h \) to any an existing subsidy pattern involves a re-allocation of subsidy over \( B \) so as to minimize the subsidy cost of capturing each borrower, subject to the PC and BC. Therefore, a Nash equilibrium must satisfy the following necessary conditions: (i) \( |\Pi_i^{MFI}| = |\Pi_j^{MFI}| \) for \( \forall i, j \in B^p \cup \widehat{B}^{NP} \) and \( h = \{1, 2\} \), or that the loss from
capturing all $n_1^* + n_2^*$ borrowers in the total portfolios of 1, 2 must be equal. (Suppose not: If 
\[ |\Pi_i^{MFI}| > |\Pi_j^{MFI}| \] and $i$ and $j$ belong to the portfolio of different MFIs, this implies that MFI$_h$ with $i$ can
release $i$ and capture $j$ from MFL$_h$ while relaxing its BC; if $i$ and $j$ both belong to MFI$_h$, then this implies
that $|\Pi_i^{MFI}|$ or $|\Pi_j^{MFI}|$ must be $> |\Pi_x^{MFI}|$, where borrower $x$ is some borrower in the portfolio of
MFL$_h$. If $|\Pi_i^{MFI}|$ or $|\Pi_j^{MFI}| > |\Pi_x^{MFI}|$, then borrower $x$ can be captured from MFL$_h$ while relaxing the BC
of MFI$_h$. If $|\Pi_i^{MFI}|$ or $|\Pi_j^{MFI}| < |\Pi_x^{MFI}|$, then either $i$ or $j$ can be captured from MFI$_h$ by MFI$_{-h}$, thus
relaxing the BC of MFI$_{-h}$.) (ii) The PC must be binding for the poorest borrower $i \in B^P \cup \tilde{B}^{NP}$, where
$i = i_k^-$. (Suppose not: If the PC is not satisfied, then this is a contradiction because $i_k^-$ cannot be in the
total MFI portfolio, $B^P \cup \tilde{B}^{NP}$. If the PC is satisfied, but non-binding for $i_k^-$, then by LEMMA 2,
borrower $i_k^- - 1$ can be captured for a lower subsidy cost than $i_k^-$ by either MFI$_h$ or MFI$_{-h}$ (assuming
sufficiently small $\xi$), thus relaxing the BC of either MFI.) (iii) The BC must bind for both 1, 2.
(Suppose not: In this case, an additional borrower can be captured by MFI$_h$ if its BC is not binding, and
$n_1^*$ and $n_2^*$ are not maximized.) The unique Nash equilibrium that satisfies all three necessary conditions
is the subsidy pattern in which $n_1^* + n_2^*$ borrowers in $B^P \cup \tilde{B}^{NP}$ receive the subsidy equal to $|\Pi_i^{MFI}|$,
i = $i_k^-$, so that $G_1 + G_2 = (n_1^* + n_2^*)|\Pi_i^{MFI}|$. The subsidy allocation is a Nash equilibrium since the
subsidy patterns by 1, 2 constitute a mutual best response; it is a unique Nash equilibrium because it is
the only subsidy pattern that satisfies the necessary conditions, (i),(ii), and (iii), for a Nash equilibrium.

Thus each borrower $i \in B^P \cup \tilde{B}^{NP}$ will receive a subsidy equal to $\frac{G_1 + G_2}{n_1^* + n_2^*}$ and MFI$_h$ will lend to
\[ \frac{G_h(n_1^* + n_2^*)}{G_1 + G_2} \] borrowers, and the share of borrowers for MFI$_h$ will be $\frac{G_h}{G_1 + G_2}$ as seen in Figure 3,
where the specific types of borrower $i$ served by each institution are undetermined. □
Proof of Proposition 5: (For simplicity, and without loss of generality, consider the case in which \( G = 0 \) for all competing MFIs.) Let the set of possible contracts \( \{ V_i, r_i \} | \alpha \), for some borrower \( i \in B^p \) satisfy \( \Pi_i^{MFI} (V_i, r_i) = 0 | \alpha \), or zero MFI profits at information-sharing level \( \alpha \). Note that the Bertrand equilibrium MFI contract \( \{ V_i^{*\ast}, r_i^{*\ast} \} | \alpha \in \{ V_i, r_i \} | \alpha \), and that by Lemma 1 the Bertrand equilibrium contract is Pareto efficient. First we show that as \( \alpha \) declines to \( \alpha' \), we have \( \Pi_i^{MFI} (V_i, r_i) < 0 \) for all \( \{ V_i, r_i \} | \alpha \).

Partially differentiating the right-hand-side of (12) we obtain
\[
\frac{\partial \Pi_i^{MFI}}{\partial \alpha} = \left[ (1 - \gamma \alpha) \left[ \gamma \tilde{\Pi} - \alpha \gamma \tilde{\Pi} - \gamma \tilde{\Pi} - \gamma \tilde{\Pi} \right] + (\alpha \gamma \alpha + \gamma) \left[ \tilde{\Pi} - \gamma \tilde{\Pi} + \gamma \tilde{\Pi} - \alpha \gamma \tilde{\Pi} \right] \right] \frac{1}{(1 - \gamma \alpha)^2}
\]
which
\[
= \gamma \alpha \left[ (\tilde{\Pi} - \gamma \tilde{\Pi} - \alpha \tilde{\Pi} - \gamma \alpha \tilde{\Pi} + \gamma \alpha^2 \tilde{\Pi} + \gamma \alpha \tilde{\Pi} + \gamma \alpha \tilde{\Pi} - \gamma \alpha^2 \tilde{\Pi}) + \gamma (1 - \gamma) (\tilde{\Pi} - \tilde{\Pi}) \right] \frac{1}{(1 - \gamma \alpha)^2}
\]
and through algebraic manipulation and cancellation of terms, reduces to
\[
\frac{\partial \Pi_i^{MFI}}{\partial \alpha} = \left[ \gamma (1 - \gamma) - \gamma \alpha (1 - \alpha) \right] \frac{(\tilde{\Pi} - \tilde{\Pi})}{(1 - \gamma \alpha)^2} > 0.
\]
Thus as \( \alpha \) falls to \( \alpha' \), we have \( \Pi_i^{MFI} (V_i, r_i) < 0 \) for all \( \{ V_i, r_i \} | \alpha \), and therefore \( \Pi_i^B (V_i^{*\ast}, r_i^{*\ast}) | \alpha \) cannot be a Bertrand equilibrium contract for borrower \( i \) at \( \alpha' \), i.e. \( \{ V_i^{*\ast}, r_i^{*\ast} \} | \alpha \in \{ V_i, r_i \} | \alpha' \). Second, we show that the Bertrand equilibrium contract \( \Pi_i^B (V_i^{*\ast}, r_i^{*\ast}) = 0 | \alpha' \) is less favorable to the borrower than \( \Pi_i^B (V_i^{*\ast}, r_i^{*\ast}) | \alpha \). Totally differentiating the left-hand-side of (12) yields
\[
\frac{dr_i}{d\alpha} = \frac{\hat{p}_a (1 + r_i - \beta)}{(1 - \hat{p}_a)} < 0,
\]
noting that \( \hat{p}_a = \frac{\partial \hat{p}}{\partial \alpha} = \left[ \gamma (1 - \gamma) - \gamma \alpha (1 - \alpha) \right] \frac{(\hat{p} - \hat{p})}{(1 - \gamma \alpha)^2} < 0 \) from differentiation of (11). Thus we know that \( \Pi_i^{MFI} (V_i^{*\ast}, r_i^{*\ast}) = 0 | \alpha' \) is characterized by a higher \( r_i \) for any given level of \( V_i \) relative to the \( \{ V_i, r_i \} | \alpha \) pair that satisfies \( \Pi_i^{MFI} (V_i^{*\ast}, r_i^{*\ast}) = 0 | \alpha \). Noting that borrower profits are monotonically decreasing in \( r_i \), i.e. that
\[
\frac{\partial \Pi_i^B}{\partial r_i} = -V_i (1 - p_i) < 0,
\]
it must be true that \( \Pi_i^B (V_i^{*\ast}, r_i^{*\ast}) | \alpha' \) is less than \( \Pi_i^B (V_i^{*\ast}, r_i^{*\ast}) | \alpha \) since under \( \alpha' \), borrower \( i \in B^p \) pays a strictly higher interest rate \( r_i \) for any level of borrowed capital \( V_i \) at \( \alpha' \) than at information-sharing level \( \alpha > \alpha' \).
References


